

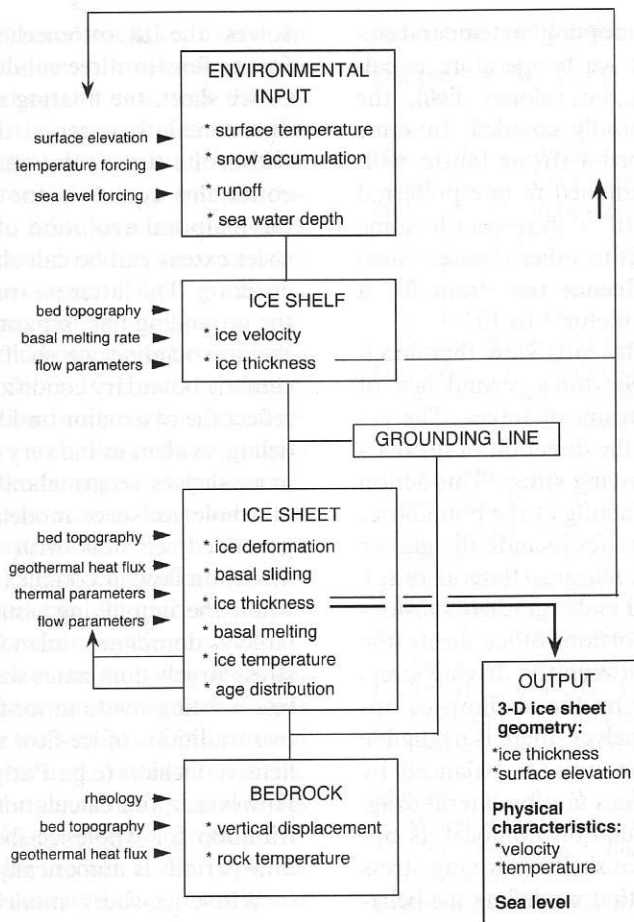
- Bentley, C. R., and J. M. Wahr. "Satellite Gravity and the Mass Balance of the Antarctic Ice Sheet." *J. Glaciol.* 44 (147) (1998): 207–213.
- Bolzan, J. F., and K. C. Jezek. "Accumulation Rate Changes in Central Greenland from Passive Microwave Data." *Polar Geography* 24 (2) (1999): 98–112.
- Budd, W. F., and R. C. Warner. "A Computer Scheme for Rapid Calculations of Balance Flux Distributions." *Ann. Glaciol.* 23 (1996): 21–27.
- Church, J. A., and J. M. Gregory. "Changes in Sea Level." In *Climate Change 2001: The Scientific Basis*, edited by J. T. Houghton, Y. Ding, D. J. Griggs, M. Noguer, P. J. van der Linden, X. Dai, K. Maskell, and C. A. Johnson. Cambridge: Cambridge University Press, 2001, pp. 639–693.
- Cuffey, K. M., and G. D. Clow. "Temperature, Accumulation, and Ice Sheet Elevation in Central Greenland Through the Last Deglacial Transition." *J. Geophys. Res.* 102 (C12) (1997): 26,383–26,396.
- Hamilton, G. S., I. M. Whillans, and P. J. Morgan. "First Point Measurement of Ice-Sheet Thickness Change in Antarctica." *Ann. Glaciol.* 27 (1998): 125–129.
- ISMSS Committee. "Recommendations for the Collection and Synthesis of Antarctic Ice Sheet Mass Balance Data." *Global and Planetary Change* 42 (2004), 1–15.
- Jacobs, S. S., H. H. Hellmer, and A. Jenkins. "Antarctic Ice Sheet Melting in the Southeast Pacific." *Geophys. Res. Letters* 23 (9) (1996): 957–960.
- Kapsner, W. R., R. B. Alley, C. A. Shuman, S. Anandakrishnan, and P. M. Grootes. "Dominant Control of Atmospheric Circulation on Snow Accumulation in Central Greenland." *Nature* 373 (1995): 52–54.
- Le Meur, E., and P. Huybrechts. "A Comparison of Different Ways of Dealing with Isostasy: Examples from Modelling the Antarctic Ice Sheet During the Last Glacial Cycle." *Ann. Glaciol.* 23 (1996): 309–317.
- Lythe, M., D. G. Vaughan, and the Bedrock Consortium. "BEDMAP: A New Ice Thickness and Subglacial Topographic Model of Antarctica." *J. Geophys. Res.* 106 (B6) (2001): 11,335–11,352.
- Mayewski, P. A., and I. D. Goodwin. *International Trans-Antarctic Scientific Expedition (ITASE)*. PAGES/SCAR Workshop Report Series, 97-1. Bern, Switzerland, 1997.
- Polar Research Board. *Glaciers, Ice Sheets, and Sea Level: Effect of a CO₂-induced Climatic Change*. Report of a Workshop held in Seattle, Washington, September 13–15, 1984. U.S. DOE/ER/60235-1, 1985.
- Rignot, E. "Mass Balance of East Antarctic Glaciers and Ice Shelves from Satellite Data." *Ann. Glaciol.* 34 (2002): 217–227.
- Rignot, E., and R. H. Thomas. "Mass Balance of Polar Ice Sheets." *Science* 297 (5586) (2002): 1502–1506.
- Shepherd, A., D. J. Wingham, and J. A. D. Mansley. "Inland Thinning of the Amundsen Sea Sector, West Antarctica." *Geophys. Res. Lett.* 29 (10) (2002): 2-1 to 2-4.
- Winebrenner, D. P., R. J. Arthern, and C. A. Shuman. "Mapping Greenland Accumulation Rates Using Observations of Thermal Emission at 4.5 cm Wavelength." *J. Geophys. Res.* 106 (D24) (2001): 33,919–33,934.
- Zwally, H. J., B. Schultz, W. Abdalati, J. Abshire, C. Bentley, A. Brenner, J. Bufton, J. Dezio, D. Hancock, D. Harding, T. Herring, B. Minster, K. Quinn, S. Palm, J. Spinhrne, and R. Thomas. "ICESat's Laser Measurements of Polar Ice, Atmosphere, Ocean, and Land." *J. Geodynamics* 34 (2002): 405–445.

ICE SHEET MODELING

Ice sheet modeling underpins much of our understanding of the Antarctic Ice Sheet. A primary motivation for developing mathematical models of ice flow is to gain better insight of the key processes controlling ice-sheet behaviour and to predict the ice sheet's response to external forcing. Modeling necessarily implies a simplified description of reality, however analytical methods can only be used for the most simple problems. Therefore, ice-dynamic models use numerical methods to solve continuous equations on a numerical grid with the aid of a computer. Ice-flow models are commonly based on fundamental physical laws and assumptions thought to describe glacier flow.

Models can be separated into two categories, namely diagnostic and prognostic models. A diagnostic model describes a certain process while a prognostic model predicts how a quantity or process evolves with time. Diagnostic ice-sheet models often isolate a small part of the ice sheet in great detail or consider the physics of a specific process in a schematic way. They are useful to highlight the importance of certain mechanisms and provide insight in key processes governing ice flow. Prognostic models mostly predict the evolution of ice thickness and thus glacier geometry over time. Such models often attempt to be comprehensive in the number of processes taken into account, however sometimes at the expense of a rigorous consideration of the full details of a particular component.

A further distinction can be made on how models embody horizontal space: either they study the dynamics of selected one-dimensional flowlines within the ice sheet or they study the ice sheet in the full two-dimensional horizontal plane. The former type is often referred to as flowline or flowband model and the latter as planform model. Planform models often average processes over the vertical extent, in which case these models are referred to as two-dimensional planform or vertically integrated models. Otherwise they incorporate vertical processes explicitly. Examples of such vertical processes are ice temperature, stress, and velocity components, as well as ice crystal fabric and water content. Such models are called three-dimensional thermomechanical models and are at the top end of the class of ice-sheet models. They are able to describe the time-dependent flow and shape of real ice sheets, and are akin to general circulation models developed in other branches of climate science. Their development closely follows technical process in such fields as computer power, ice-core and sediment drilling, remote sensing, and geophysical dating techniques, which are both providing the required calculating means and the necessary data to feed and validate such models.



Structure of a comprehensive three-dimensional ice-sheet model applied to the Antarctic ice sheet. The inputs are given at the left-hand side. Prescribed environmental variables drive the model, which has ice shelves, grounded ice, and bed adjustment as major components. The position of the grounding line is not prescribed, but internally generated. Ice thickness feeds back on surface elevation, an important parameter for the calculation of the mass balance. The model essentially outputs the time-dependent ice-sheet geometry and the coupled temperature and velocity fields. (From Huybrechts 2004.)

Historically, planform time-dependent modeling of ice sheets largely stems from early work by Mahaffy (1976) and Jenssen (1977), extending on the pioneering "Derived Physical Characteristics of the Antarctic Ice Sheet" of W. F. Budd and colleagues at the Australian National Antarctic Research Expeditions published in 1971. These landmark studies introduced many concepts and techniques that are still used in glaciology today. The most important concept made use of the fact that the horizontal extent of an ice sheet is large compared with its thickness. In what became known as the shallow-ice approximation (Hutter 1983), longitudinal derivatives of stress, velocity, and temperature are assumed small compared to vertical derivatives. This greatly simplifies the numerical solution. Although the assumption is only fully satisfied over inland portions of continentally based ice, it has shown general applicability in large-scale ice-sheet modeling as long as surface slopes are evaluated over horizontal distances at least an order of magnitude greater than ice thickness.

The core of an ice-sheet model calculates how ice flows downhill in response to stresses set up by gravity. This ice flow results from internal deformation and from ice sliding over its bed where the basal temperature has reached the melting temperature and a lubricating water-saturated layer has formed. Whereas basal sliding depends to a large extent on the properties of the bed under the ice, internal deformation is the inherent manifestation of individual ice crystals subjected to stress. This deformation is reasonably well understood on the macro scale and can be reliably modelled taking into account Glen's flow law. That is an empirical relation derived from laboratory tests, which is most commonly used in ice flow modeling. It considers ice as a nonlinear viscoelastic fluid, relating strain rates to stresses raised mostly to the third power. The rate of deformation for a given stress also depends on the temperature of the ice and the fabric of the ice. The warmer the ice, the easier it deforms. For the temperature range encountered in the Antarctic Ice Sheet, three orders of magnitude are involved. In the flow law, this temperature effect

is usually incorporated by adopting a temperature-dependent rate factor. If the ice temperature is calculated simultaneously with the velocity field, the flow is called thermomechanically coupled. In some instances, the ice has developed a strong fabric, with the majority of crystal axes aligned in one preferred direction, making the ice "soft" with respect to some stress and "hard" with respect to other stresses. Such fabric development may influence the strain for a given stress by an additional factor 3 to 10.

Because glacier flow is sufficiently slow that accelerations can be neglected, Newton's second law of motion reduces to an equilibrium of forces. The action force making ice flow in the direction of decreasing surface elevation is the driving stress. This action is opposed by resistive forces acting at the boundaries of the ice mass. These boundaries include the glacier bed (basal drag), the lateral margins (lateral drag), and the up- and down-glacial ends (gradients in longitudinal stress). In interior portions of ice sheets, the force balance is essentially between the driving stress and basal drag as predicted by the shallow-ice approximation. In floating ice shelves, there is negligible basal friction and the driving stress is balanced by gradients in longitudinal stresses and by lateral drag. This makes the velocity calculation nonlocal as opposed to inland ice flow. In ice shelves, driving stress is balanced more broadly, so that modelling the behaviour at any point requires knowledge of all of the surrounding stresses affecting the ice mass. Lateral drag and longitudinal stress gradients also play an important role in the fast outlet glaciers and ice streams that are responsible for the bulk of the ice discharge towards the margin. As such they represent a transitional region between inland-ice and ice-shelf dynamics. Because of the low driving stresses in the downstream portions of such ice streams, much, if not all, of the differential flow between the ice surface and the bedrock is caused by either basal sliding or by deformation of a subglacial mud (till) layer. Fast-glacier conditions at the base are, however, poorly understood. Processes related to bed roughness, till rheology, and basal water pressure are all thought to be important elements, but a realistic basal boundary condition for use in numerical models has not yet been developed.

Based on these principles, the advent of bigger and faster computers has allowed elaborate numerical models of the Antarctic Ice Sheet to be constructed. At the heart of such a model is the simultaneous solution of two evolutionary equations for ice thickness and temperature, together with diagnostic representations of the ice velocity components. These express fundamental conservation laws for momentum, mass, and heat, supplemented with Glen's flow law for polycrystalline ice deformation. The model

solves the thermomechanically coupled equations for ice flow in three subdomains, namely the grounded ice sheet, the floating ice shelf, and a stress transition zone in between at the grounding line. The flow within the three subdomains is coupled through the continuity equation for ice thickness, from which the temporal evolution of ice sheet elevation and ice sheet extent can be calculated by applying a flotation criterion. The latter treatment allows for migration of the grounding line, separating the land-based ice from the surrounding ice shelf, in response to changes in climatic boundary conditions. The various subdomains reflect the two major traditions of ice-deformation modeling, evident in the very different physical conditions in ice shelves versus inland ice. An important difficulty in whole ice-sheet models lies with the coupling of grounded ice flow with floating ice flow and with modeling flow in complex regions such as ice streams, where the simplifying assumption that one shear stress largely dominates inland flow and one stretching stress largely dominates shelfy flow breaks down. Progress is being made in ice-flow models to combine the two traditions of ice-flow modeling in a more comprehensive fashion (e.g., Pattyn 2003; Payne et al. 2004). However, a full calculation of the complete stress distribution for whole-ice-sheet integrations over longer time periods is numerically not yet feasible.

Whole-ice-sheet modeling of the Antarctic Ice Sheet further involves simulation of surface mass fluxes (snow fall, wind drift, sublimation, melting followed by runoff or refreezing), sinking or rising of the underlying bedrock in response to changing ice load, heat transfer under the ice and into the bedrock affecting melting/frozen regions and the deformation rate of ice, interactions of ice shelves with the ocean, and more. Interaction with the atmosphere and the ocean in large-scale Antarctic Ice Sheet models is carried out by prescribing the climatic input, consisting of the surface mass balance (accumulation minus ablation, if any), the surface temperature, and the basal melting rate below the ice shelves. Changes in these fields are often heavily parametrized in terms of air or ocean temperature but can also be derived from calculations with atmosphere and ocean models. Models of this type are usually forced by time series of regional temperature changes (available from ice-core studies) and by the eustatic component of sea-level change, relative to present values.

Three-dimensional ice sheet models are typically implemented using finite-difference techniques on a regular grid of nodes in the two horizontal dimensions, and using a stretched coordinate system in the vertical. Horizontal grid resolutions are mostly in the range of 10 to 50 km with between 20 and 100 layers in the vertical, concentrated towards the base

where the bulk of the deformation takes place. In finite-difference models, gradients of continuous functions are obtained by dividing the values of the parameter at grid points by the distance between the grid points. Finite element implementations also exist but these are often restricted to a smaller domain. An advantage of the finite element method is that the element size can be reduced in areas of high gradients and increased in areas of low gradients. Furthermore, element shapes can be adjusted to conform to boundaries that would otherwise be awkward to model with rectangular elements. However, changing spatial patterns over time and varying ice-sheet domains have proven to be a challenge to the popularity of finite-element methods in glaciological modeling. Recent Antarctic model studies have benefited from much improved compilations of crucial input data such as bed elevation, surface elevation, and ice thickness that became available on high-resolution grids from the BEDMAP project (Lythe et al. 2001).

Three-dimensional models of the Antarctic Ice Sheet have been used to examine mechanisms and thresholds of ice-sheet inception during the Tertiary (DeConto and Pollard 2003), the expansion and contraction of the Antarctic Ice Sheet during the glacial-interglacial cycles (Huybrechts 2002), and the likely effects of greenhouse-induced polar warming (Huybrechts et al. 2004). In this context, the key interactions being investigated are between the effects of a change in climate on the accumulation and ablation fields and the ice sheet's response in terms of changed geometry and flow, including the ice sheet's contribution to the worldwide sea level stand. Related work considers the Antarctic Ice Sheets as a boundary condition for other components of the Earth's geophysical system, providing changes in surface loading for isostasy and gravity models, or providing changes in freshwater fluxes for ocean models. Thermomechanical ice-sheet models are also being used to investigate the potential for internally generated flow instabilities, especially concerning the West Antarctic Ice Sheet, or explain recently detected accelerations of outlet glaciers taking into account higher-order stress calculations (Payne et al. 2004). The general appreciation is that current models available to the community perform best for the largely continental-based East Antarctic Ice Sheet; however, many challenges pertain to the modelling of the marine-based West Antarctic Ice Sheet.

PHILIPPE HUYBRECHTS

See also **Antarctic Ice Sheet: Definitions and Description; Climate; Climate Change; Earth System, Antarctica as Part of; Glaciers and Ice Streams; Ice Ages; Ice-Rock Interface; Ice Sheet Mass Balance; Ice Shelves; Icebergs**

References and Further Reading

- DeConto, R. M., and D. Pollard. "Rapid Cenozoic Glaciation of Antarctica Induced by Declining Atmospheric CO₂." *Nature* 421 (2003): 245–249.
- Hooke, R. L. *Principles of Glacier Mechanics*. 2nd ed. Cambridge: Cambridge University Press, 2005.
- Hutter, K. *Theoretical Glaciology*. Dordrecht, Netherlands: D. Reidel, 1983.
- Huybrechts, P. "Sea-Level Changes at the LGM from Ice-Dynamic Reconstructions of the Greenland and Antarctic Ice Sheets During the Glacial Cycles." *Quaternary Science Reviews* 21 (1–3) (2002): 203–231.
- Huybrechts, P. "Antarctica: Modelling." In *Mass Balance of the Cryosphere: Observations and Modelling of Contemporary and Future Changes*, edited by J. L. Bamber and A. J. Payne. Cambridge: Cambridge University Press, 2004, pp. 169–225.
- Huybrechts, P., J. M. Gregory, I. Janssens, and M. Wild. "Modelling Antarctic and Greenland Volume Changes During the 20th and 21st Centuries Forced by GCM Time Slice Integrations." *Global and Planetary Change* 42 (2004): 83–105, doi:10.1016/j.gloplacha.2003.11.011.
- Jenssen, D. "A Three-Dimensional Polar Ice Sheet Model." *Journal of Glaciology* 18 (1977): 373–389.
- Lythe, M., and D. G. Vaughan. "BEDMAP: A New Ice Thickness and Subglacial Topographic Model of Antarctica." *Journal of Geophysical Research* 106 (B6) (2001): 11,335–11,352.
- Mahaffy, M. A. W. "A Three-Dimensional Numerical Model of Ice Sheets: Tests on the Barnes Ice Cap, Northwest Territories." *Journal of Geophysical Research* 81 (1976): 1059–1066.
- Paterson, W. S. B. *The Physics of Glaciers*. 3rd ed.. Oxford: Pergamon, 1994.
- Pattyn, F. "A New Three-Dimensional Higher-Order Thermomechanical Ice Sheet Model: Basic Sensitivity, Ice Stream Development, and Ice Flow Across Subglacial Lakes." *Journal of Geophysical Research* 108 (2382) (2003), doi:10.1029/2002JB002329.
- Payne, A. J., A. Vieli, A. Shepherd, D. J. Wingham, and E. J. Rignot. "Recent Dramatic Thinning of Largest West Antarctic Ice Stream Triggered by Oceans." *Geophysical Research Letters* 31 (2004), L23401, doi:10.1029/2004GL021284.
- Van der Veen, C. J. *Fundamentals of Glacier Dynamics*. Rotterdam/Brookfield: A. A. Balkema, 1999.

ICE SHELVES

General Characteristics

Ice shelves are the floating parts of an ice sheet. They form at the margins where the ice sheet becomes thin enough to float free of a bed that lies below sea level, allowing seawater to circulate beneath the ice. Ice-shelf-like features also form in the interior of ice sheets where the ice floats on subglacial lakes. Dynamically, ice shelves are distinct from other parts of the