Comparison of Preconditioning Techniques for Optimization of a Nonhydrostatic, Parallel Tsunami Simulation Model

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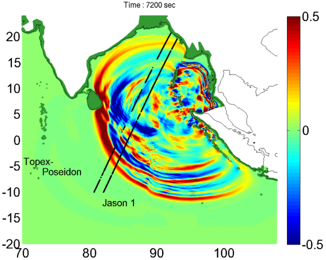
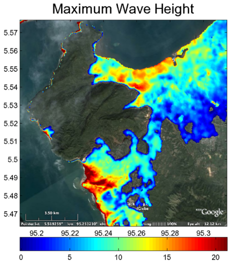
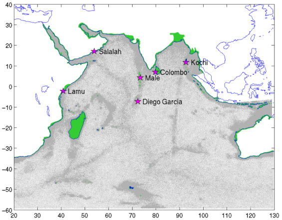
Overview

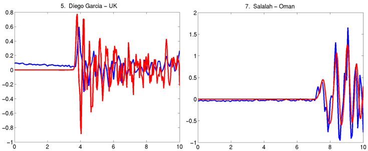
TsunAWI



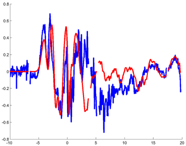




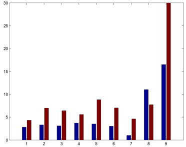
Tide Gauges



Jason 1



Inundation



figures by S. Harig

Shallow Water Model

Depth-integrated mass and momentum equation

$$\eta_t + \nabla \cdot (\tilde{\mathbf{u}} H) = 0, \qquad (1)$$

$$\tilde{\mathbf{u}}_t + (\tilde{\mathbf{u}} \cdot \nabla)\tilde{\mathbf{u}} + \mathbf{f} \times \tilde{\mathbf{u}} + g\nabla\eta + \mathbf{F} = 0, \qquad (2)$$

with surface water elevation η , horizontal velocity $\tilde{\mathbf{u}} = (\tilde{u}, \tilde{v})$ as unknowns.

Initial Conditions:

$$\eta|_{t=0} = \eta_0, \quad \forall (x,y) \in \Omega$$

$$\tilde{\mathbf{u}}|_{t=0} = 0, \quad \forall (x,y) \in \Omega$$

Boundary Conditions:

$$\tilde{\mathbf{u}} \cdot \mathbf{n} = \begin{cases} \sqrt{\frac{g}{H}}\eta, & \forall (x,y) \in \Gamma_{ob} \\ 0, & \forall (x,y) \in \Gamma_{sb} \end{cases}$$

Nonhydrostatic Correction Terms

- Idea: nonhydrostatic model = hydrostatic model + nonhydrostatic correction (R. Walters, 05)

$$\mathbf{u}^{n+1} = \tilde{\mathbf{u}}^{n+1} - \Delta t \nabla q^{n+1} - \Delta t \frac{q^{n+1}}{H^n} \nabla(\eta^{n+1} - h), \qquad (3)$$

$$w_\eta^{n+1} = \tilde{w}_\eta^{n+1} + 4\Delta t \frac{q^{n+1}}{H^n}. \qquad (4)$$

with hydrostatic velocity $(\tilde{\mathbf{u}}, \tilde{w})$, nonhydrostatic bottom pressure $q = q_{-h}$ and total water depth $H = \eta + h$.

Initial Condition: $\tilde{w}|_{t=0} = 0$,
Boundary Conditions: $q_\eta = 0, w_{-h} = -\mathbf{u} \cdot \nabla h$.

Additional work

- Computation of w_η and w_{-h} by FEM based systems of linear equations.
- Inclusion of the correction eqations in the integral continuity equation

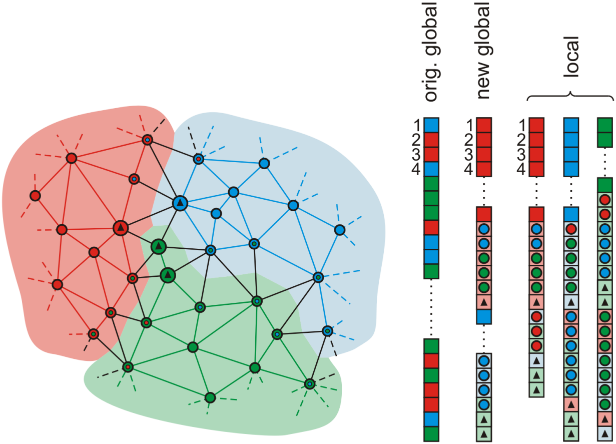
$$\int \phi(\nabla \cdot \mathbf{u}(q) + \delta_z w(q))\mathrm{d}V = 0. \tag{5}$$

- Partial integration results in the system of linear equations

$$\mathbf{Aq} = \mathbf{b}.$$

MPI version

- Model runs on parallel machines via MPI communication.
- Global and local resorting by seperation of interior and interface nodes.

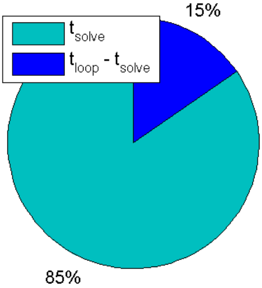


Overview

Systems of Linear Equations

- Solving the linear systems of equations takes up the most percentage of computing time.

- The mass matrices of $\tilde{w}_\eta$ and $\tilde{w}_{-h}$ are replaced by *lumped matrices*, so the vertical velocity can be solved explicitly.
- Following observations are with regard to the examination of bottom pressure q.

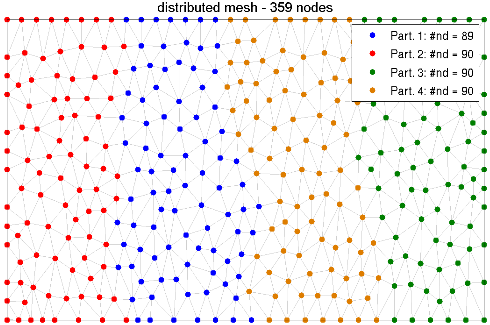


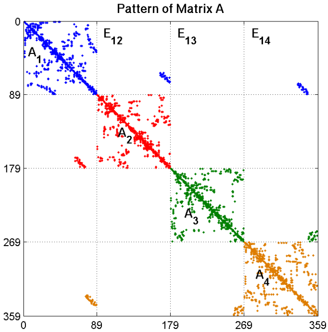
Krylov Subspace Method GMRES

- GMRES - **G**eneralized **M**inimal **RES**idual Method
- Iterative method to minimize the norm of the residual $r_i := b - Ax_i$, with $x_i \in x_0 + \text{span}\{r_0, Ar_0, A^2r_0, ..., A^{i-1}r_0\}$.
- For saving memory resources GMRES(30) is used.
- Convergence behaviour depends on properties of matrix A.
- pARMS 3.2 (Li, Saad, Sosonika)

Domain Decomposition

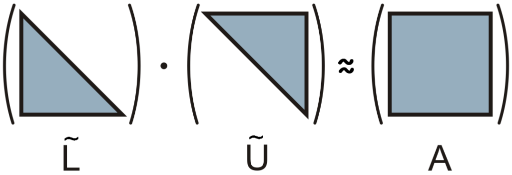
- using METIS 4.0 (G. Karypis and V. Kumar) as partitioner





Incomplete LU Factorization (ILU)

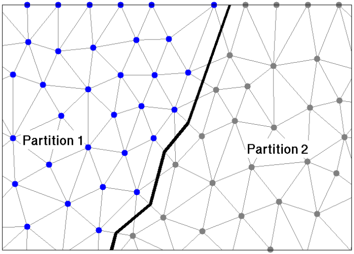
- An incomplete LU Factorization only approximates the matrix $\tilde{L}\tilde{U} \approx A$ but the triangular matrices $\tilde{L}$ and $\tilde{U}$ are sparse.
- There are several approaches to force the sparsity. Here ILU(2), ILU(3) and ILUT (pARMS 3.2) are used.

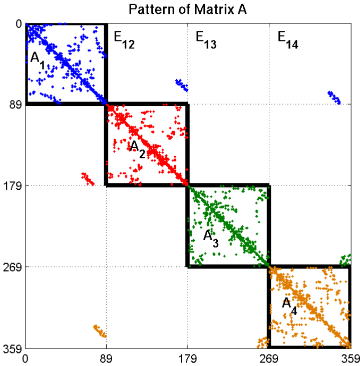


Block Jacobi (BJ)

- $A_i xi + \sum E_{ij} x_j = b_i$

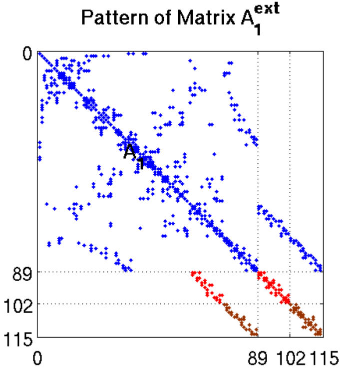
- The local preconditioner operates on the local diagonal block A_i.
- Offdiagonal blocks E_{ij} are ignored.
- No communication is required.

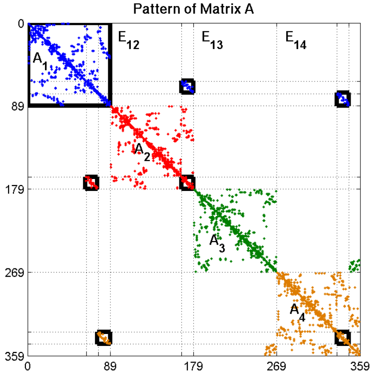




Restricted Additive Schwarz (RAS)

- Communication of values at interface nodes.
- The extended matrix A_i^{ext} is submitted to an ILU Factorization.





Schur Complement Based Preconditioners 1/2

- Seperation of local interior nodes u_i and interface nodes v_i.

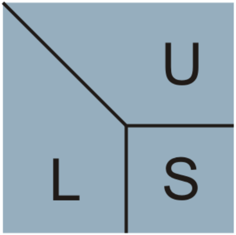
$$\underbrace{\begin{pmatrix} B_i & F_i \\ E_i & C_i \end{pmatrix}}_{A_i} \underbrace{\begin{pmatrix} u_i \\ v_i \end{pmatrix}}_{x_i} + \begin{pmatrix} 0 \\ \Sigma_j E_{ij} v_j \end{pmatrix} = \underbrace{\begin{pmatrix} f_i \\ g_i \end{pmatrix}}_{b_i} \qquad (6)$$

$$\Downarrow$$

$$u_i = B_i^{-1}(f_i - F_i v_i) \qquad (7)$$

$$S_i v_i + \sum_j E_{ij} v_j = g_i - E_i B_i^{-1} f_i \qquad (8)$$

with Schur Complement $S_i = C_i - E_i B_i^{-1} F_i$.



- if $\tilde{L}_i \tilde{U}_i \approx A_i$ then $\tilde{L}_i^S \tilde{U}_i^S \approx S_i$.

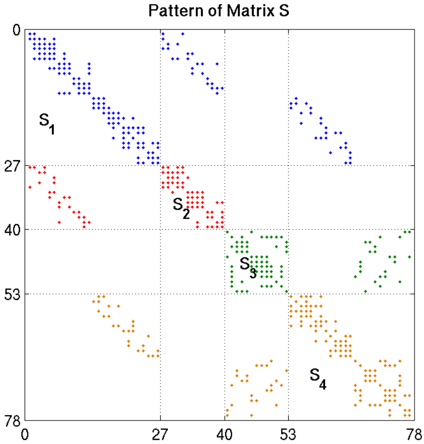
Schur Complement Based Preconditioners 2/2

approach 1 (Schur):

- ILU Factorization on S_i
- Solve (8) by inner GMRES

approach 2 (SchurRAS):

- Build global Schur matrix S.
- RAS acts on S.

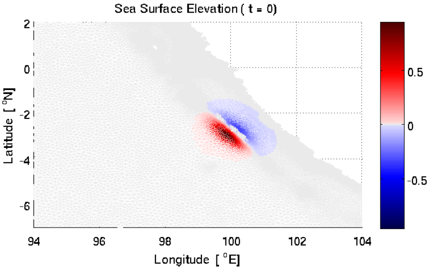


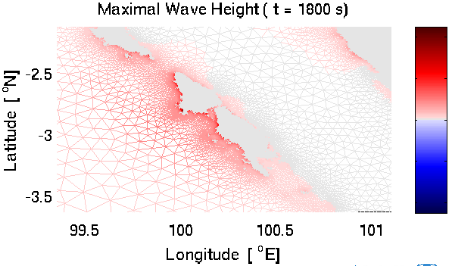
Overview

Tsunami Simulation Off the Coast of Sumatra

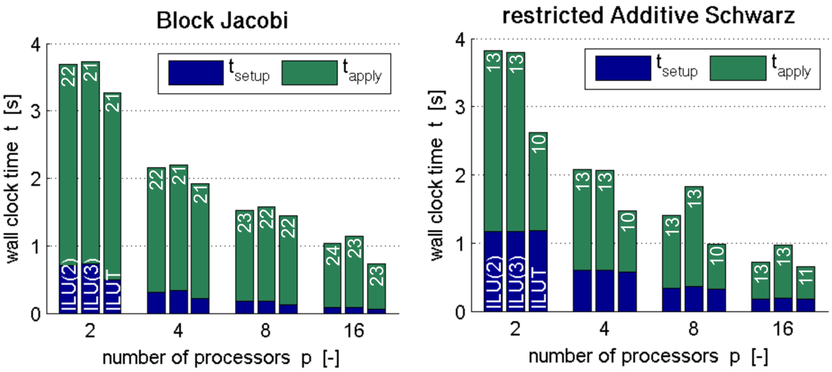
- Tsunami is initiated by an earthquake with magnitude $M_w = 7.8$.

# nodes	629061
# elements	1256019
Δt	1.0s
# timesteps	1800

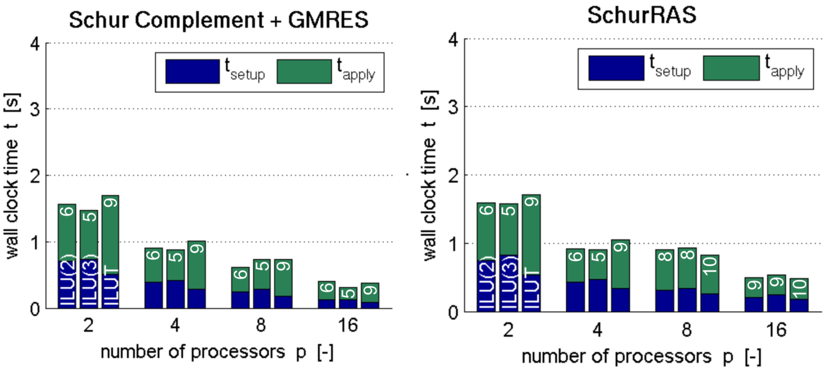




Results: BJ - RAS



Results: Schur - SchurRAS



Conclusion & Outlook

Conclusion

- The influence of the chosen preconditioning technique is not small.
- For tsunami simulation we will prefer Schur Complement based techniques.

Outlook

- Investigation of these techniques applied to more complex scenarios.
- Using the experience for other models like FESOM.