Validation of the enthalpy method by means of analytical solution

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Introduction

The large ice sheets in Greenland and Antarctica have a polythermal structure. They are mainly cold with a temperate layer at the base. In temperate ice the heat generated by viscous deformation does not increase the temperature, but causes melting. The liquid water inclusions (moisture) make this ice considerably softer than cold ice, resulting in a strong relationship between viscosity and water content (Duval, 1977; Lliboutry and Duval, 1985). The importance of this feature for ice dynamics is obvious, especially for temperate ice at the base where stresses are highest.

The enthalpy scheme presented in Aschwanden et al. (2012) describes temperature and water content in a consistent and energy conserving formulation.

Here we present two numerical experiments to test the implementation of the enthalpy scheme in numerical ice sheet models. The proposed experiments are chosen in a way that they can be conducted by numerical models with no or only minor modifications of the source codes necessary.

Used models

- Tim-FD^3 (finite-differences, Kleiner & Humbert, 2013)
- ISSM (finite-elements, e.g. Seroussi et al., 2013, http://issm.jpl.nasa.gov/)
- COMice (finite-elements, e.g. Rückamp et al., 2010, http://www.comsol.com/)

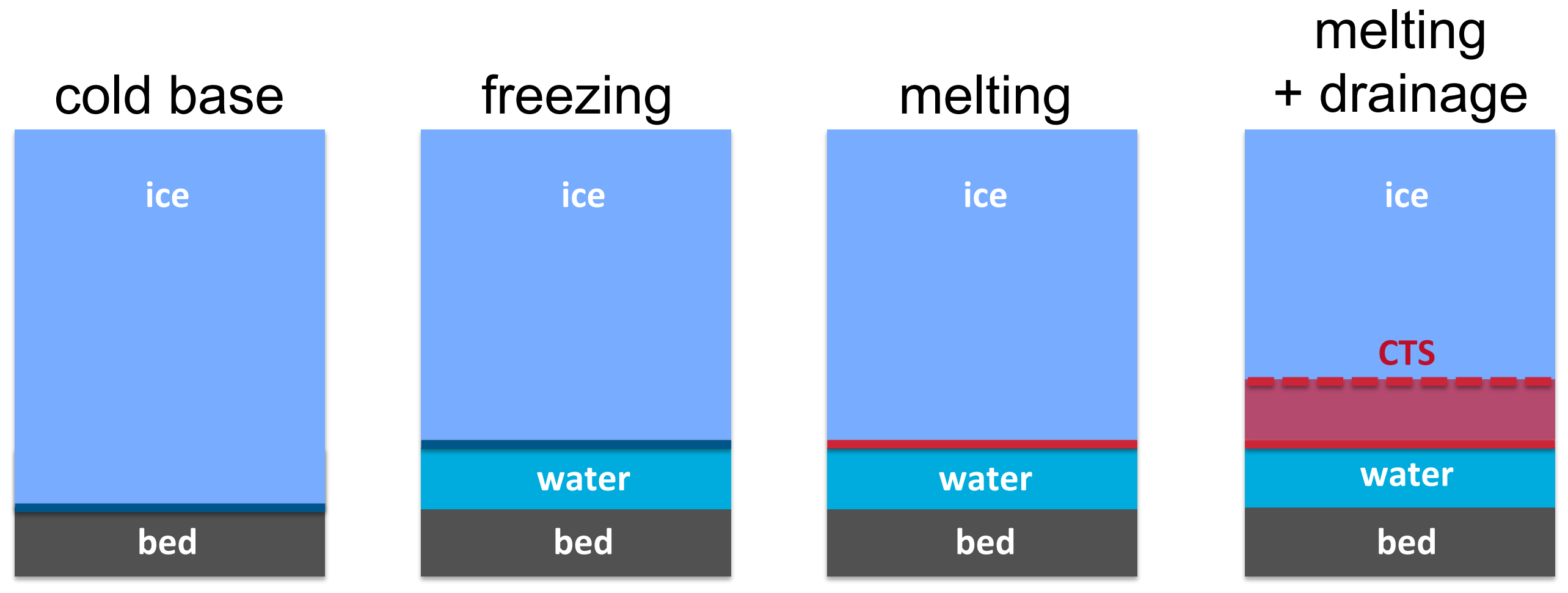
Enthalpy method

With the enthalpy method described in Aschwanden et al. (2012), the temperature T and moisture content ω are diagnostically computed from the enthalpy field E. The enthalpy field equation for the mixture of ice and liquid water depends on whether the mixture is cold or temperate. We have advection of heat, sensible heat flux in the cold ice part and sensible plus latent heat flux in temperate ice part as well as heat by internal deformation (strain heating).

$$\rho_{\mathrm{i}}\left(\frac{\partial E}{\partial t}+\mathbf{v}\nabla E\right)=\nabla\cdot\left\{\left(\begin{matrix}K_{\mathrm{c}}\nabla E\\ k_{\mathrm{i}}\nabla T_{\mathrm{pmp}}+K_0\nabla E\end{matrix}\right)\right\}+\Psi$$

Basal boundary conditions

The decision chart for the basal conditions given in Aschwanden et al. (2012) encompasses four different situations that need to be evaluated at every time step:



$-K_{\mathrm{c}}\nabla E\cdot\mathbf{n}_{\mathrm{b}}=q_{\mathrm{geo}}$ $E=E_{\mathrm{pmp}}$ $E=E_{\mathrm{pmp}}$ $-K_0\nabla E\cdot\mathbf{n}_{\mathrm{b}}=0$

Experiment 1: Parallel sided slab (transient)

The first experiment tests particularly the functionality of the boundary condition scheme and the basal melt rate calculation during transient simulations. A parallel sided slab of ice of constant thickness H=1000m is considered. The velocity and consequently the associated strain heating is zero. The geothermal heat flux at the base is constant. The surface is parallel to the bed and has zero inclination. We impose periodic boundary conditions at the sides of the block. Hence the horizontal extension does not play a role and the set-up is basically 1D (vertical). The experiment is as follows:

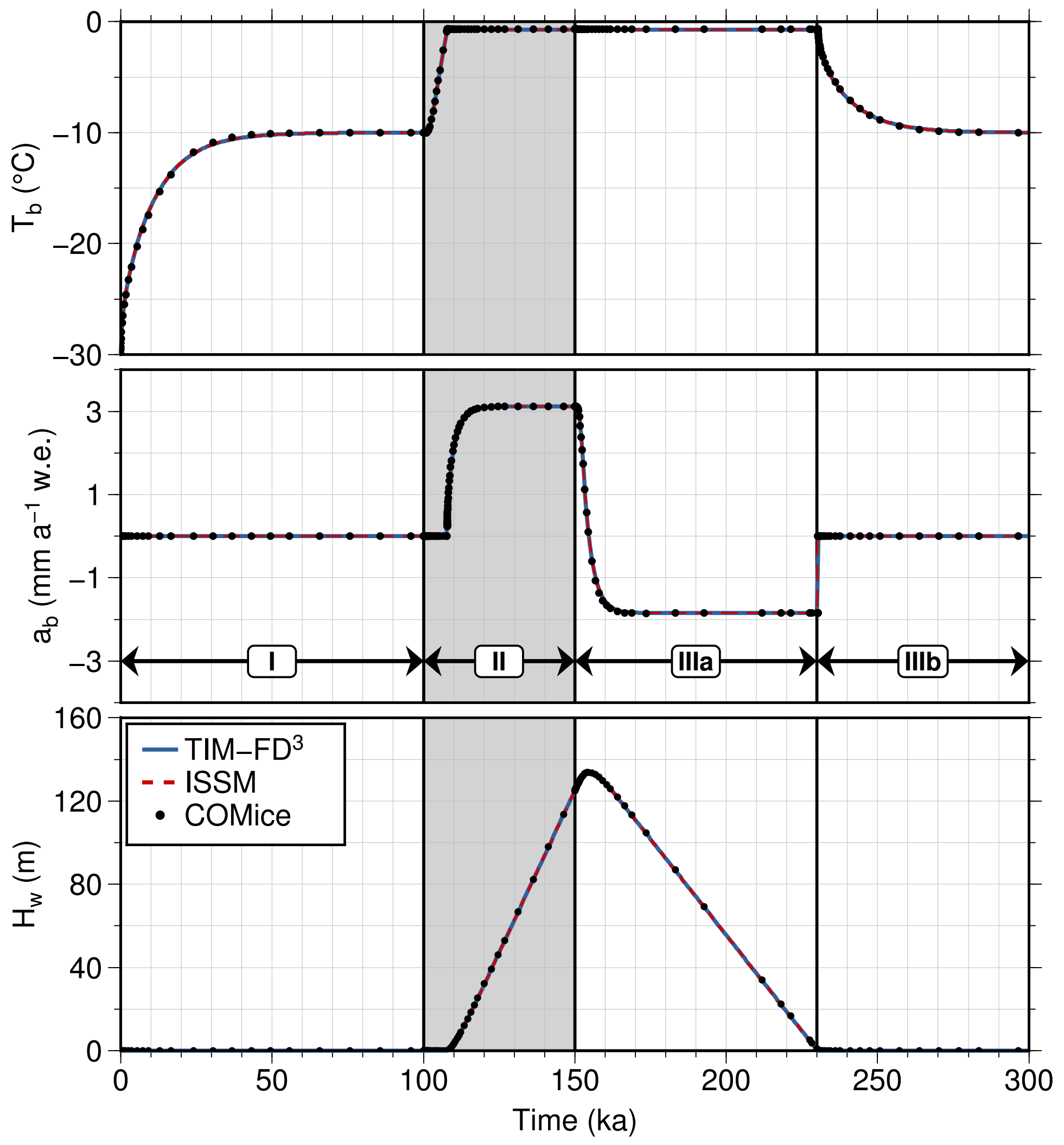
I. **Initial phase:** Starting under cold conditions with an imposed surface temperature and an initial temperature of -30°C the simulation is running for 100 ka.

II. **Warming phase:** The surface temperature is switched to -10°C and the simulation is continued for another 50 ka.

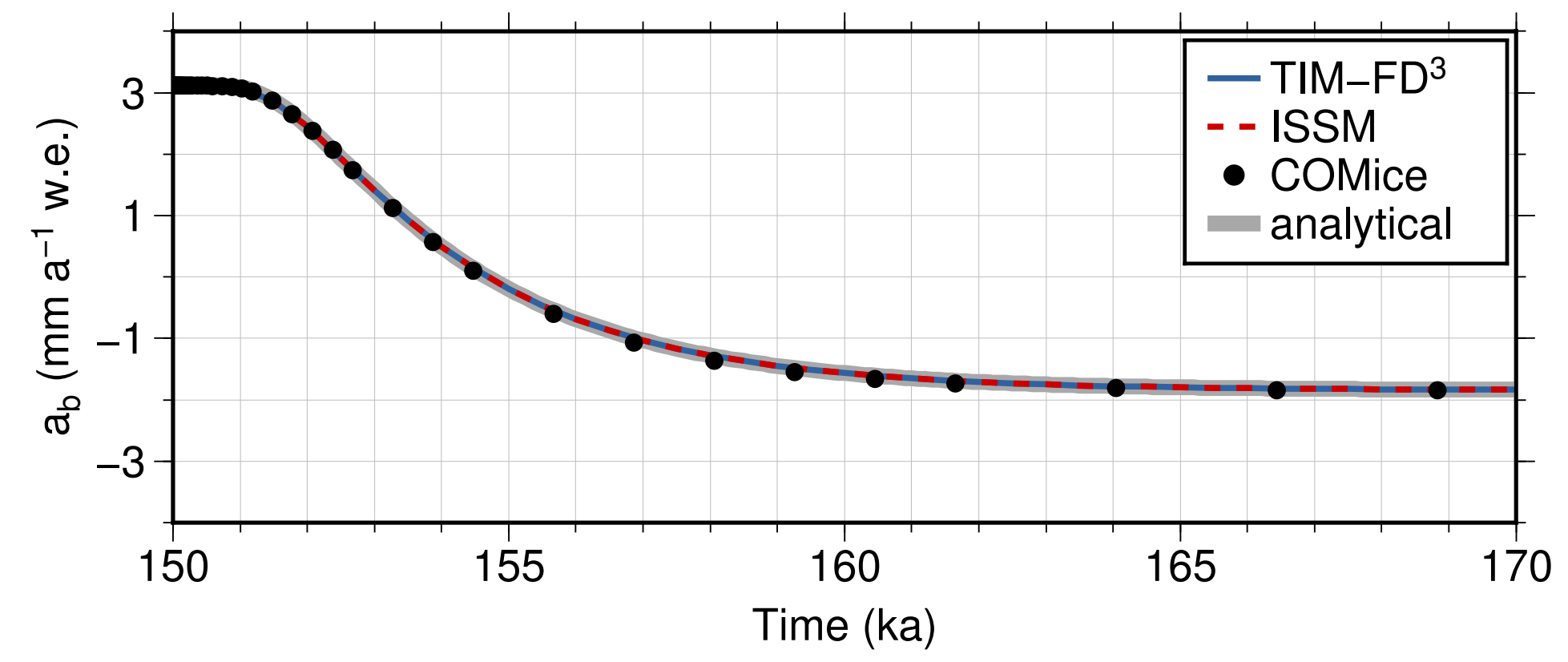
III. **Cooling phase:** The surface temperature is switched back to the initial value of -30°C and the simulation is continued for further 150 ka.

As heat conduction is the only process of heat transfer, the vertical enthalpy profiles are linear in steady-state, which is asymptotically reached at the end of each phase.

Results



Basal temperature T_b, basal melt rate a_b and basal water layer thickness H_w simulated with TIM-FD^3 (blue), ISSM (red) and COMice (black) in Exp.1. The warming phase (II) is shaded in grey.

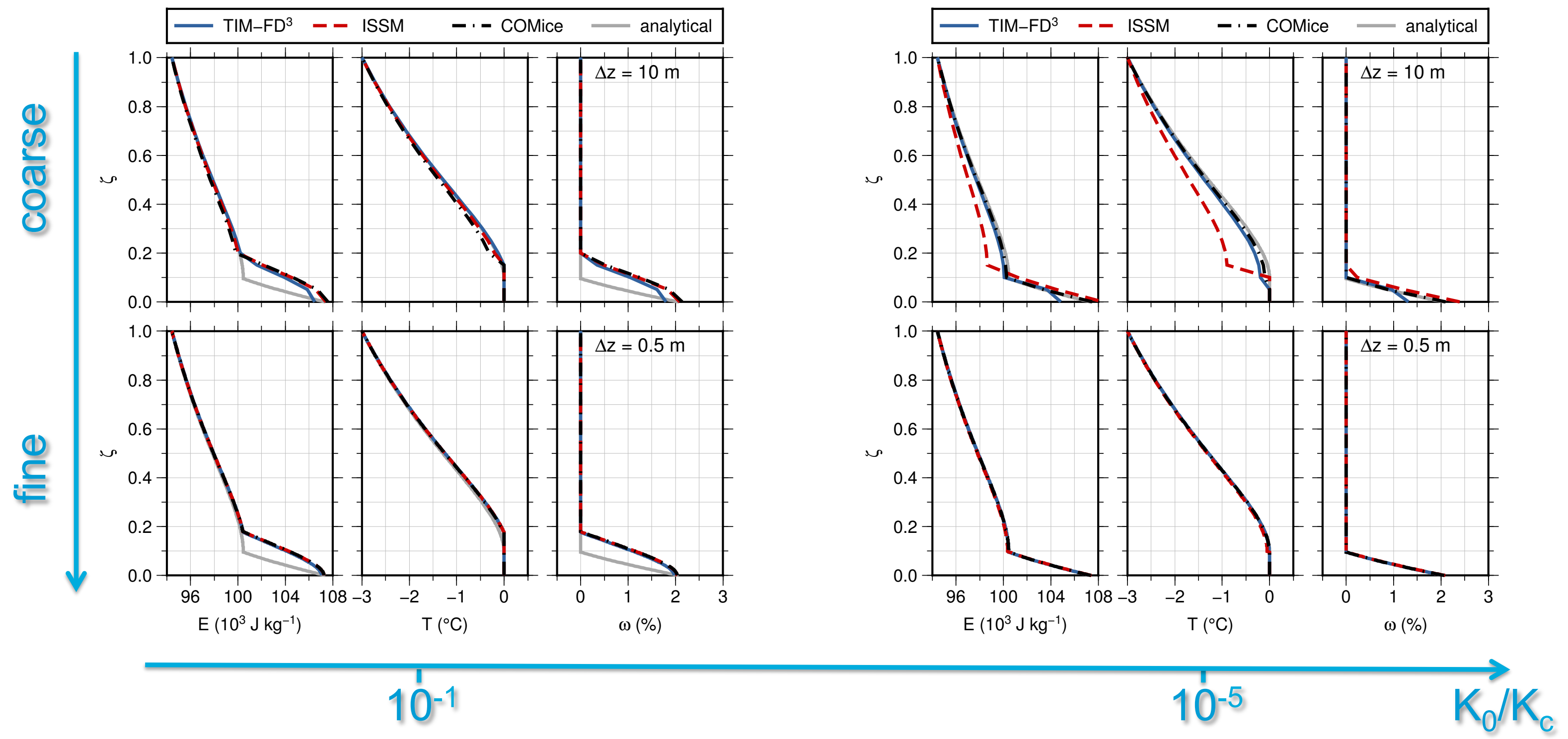


Simulation results of the basal melt rate a_b compared to the analytical solution (grey line) for the first 20 ka of the cooling phase (IIIa). The analytical solution can be found by separation of variables and Fourier analysis.

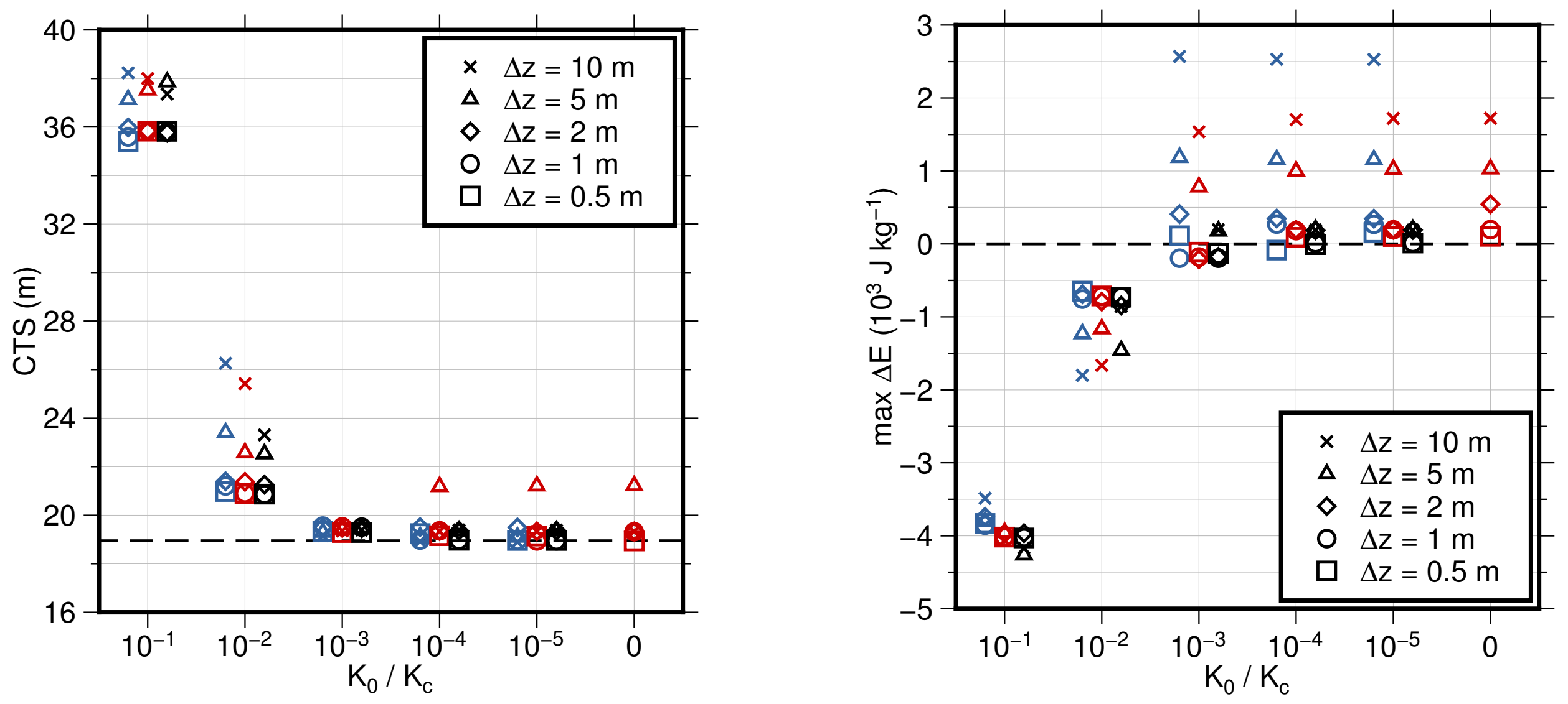
Experiment 2: Polythermal slab (steady-state)

The second experiment addresses the steady-state enthalpy profile and the resulting position of the cold-temperate transition surface (CTS). Here we apply the "parallel-sided polythermal slab" set-up with melting conditions at the CTS as given in e.g. Greve & Blatter (2009). A slab of constant ice thickness H=200m and a constant surface and bed inclination of 4° in x-direction is considered. Ice flow is decoupled from the thermal quantities by using a constant flow rate factor. The velocity throughout the ice column is prescribed (T_s=-3°C, v_z = 0.2 ma^{-1}). Model results are compared with the analytical solution described in Greve & Blatter (2009) for K_0=0.

Results



Simulated steady-state profiles compared with the analytical solution.



Simulated steady-state CTS position (left) and maximum difference to the analytical solution (right) for TIM-FD^3 (blue), ISSM (red) and COMice (black).

Conclusion

- The models (TIM-FD^3, ISSM, COMice) are able to perform the proposed experiments successfully and agree to the analytical solutions.
- For melting conditions at the CTS enthalpy scheme determines the CTS position correctly without the need of tracking the CTS explicitly and applying additional conditions at this internal boundary.
- There is a clean demand for an empirical determination of the temperate ice conductivity K_0 and an improved description of the temperate ice rheology.

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