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Antarctica: modelling

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13.1 Introduction

Mathematical modelling represents a vital tool for understanding and predicting the current and future behaviour of the Antarctic ice sheet. Above all, modelling tries to overcome the limitations of space and time associated with making direct observations. The dynamical timescales associated with many components of the Antarctic ice sheet are far larger than the limited period for which measurements are available. Models also generate information over the entire ice sheet and can yield insight into many processes that are often inaccessible for direct observation such as at the ice-sheet base. In addition, models are the only tools we have at our disposal to forecast the future evolution of the ice sheet.

Today, the Antarctic ice sheet contains 89% of global ice volume, or enough ice to raise sea level by more than 60 m (Table 13.1). Hence, only a small fractional change of its volume would have a significant effect on the global environment. The average annual solid precipitation falling onto the ice sheet is equivalent to 5.1 mm of sea level, this input being approximately balanced by ice discharge into floating ice shelves, which experience melting and freezing at their underside and eventually break up to form icebergs.

Changes in ice discharge generally involve response times of the order of 10^2 to 10^4 years. These timescales are determined by isostasy, the ratio of ice thickness to yearly mass turnover, processes affecting ice viscosity and physical and thermal processes at the bed. Hence it is likely that the Antarctic ice sheet is still adjusting to its past history, in particular to those changes associated with the last glacial–inter-glacial transition. Its future behaviour therefore has a component resulting from past climate changes as well as one related to present and future climate changes. To assess the future response of the ice sheet correctly, it is thus necessary to be able to distinguish between the long-term background trend and the anthropogenically induced signal due to recent and future climate changes.

Several methods have been used to assess the current evolution of the Antarctic ice sheet. The traditional method is to estimate the individual mass balance terms and make the budget. According to the Third Assessment Report of the Intergovernmental Panel on Climate Change (IPCC) (Church *et al*., 2001), presently available data do not allow us to

Mass Balance of the Cryosphere: Observations and Modelling of Contemporary and Future Changes, eds. Jonathan L. Bamber and Antony J. Payne. Published by Cambridge University Press.

Table 13.1. *Physical characteristics of the Antarctic ice sheet.*

Area (10^6 km^2)	12.37
Volume (10^6 km^3)	25.71
Volume (sea-level equivalent, m)[a,b]	61.1
Accumulation[c] (10^{12} kg per year)	1843 ± 76
Accumulation (sea-level equivalent, mm per year)[c]	5.1 ± 0.2
Runoff[c] (10^{12} kg per yr)	10 ± 10
Iceberg discharge (10^{12} kg per year)	2072 ± 304
Mass turnover time (years)	≈15 000

[a] Assuming an oceanic area of 3.62×10^8 km^2.
[b] After isostatic rebound and sea-water replacing grounded ice.
[c] Grounded ice only.
Sources: Huybrechts *et al.* (2000) and Church *et al.* (2001).

constrain this budget to better than $-376 \pm 384 \times 10^{12}$ kg per year ($-16.7\% \pm 17.1\%$ of total mass input), not significantly different from zero. Satellite altimetry has great potential to estimate the current trend, but is presently hampered by incomplete coverage and records that are too short to distinguish confidently between a short-term mass balance variation and the longer-term ice-sheet dynamic imbalance.

Because of the inaccuracy of the budget method and the short duration of satellite records, modelling of the entire ice-sheet–bedrock system over time may fill an important gap. It can provide an alternative approach to the balance problem by simulating the evolution of the ice sheet and its underlying bed over a sufficiently long time to remove transient effects, and subsequently analysing the imbalance patterns which result for the present day. Apart from yielding more insight into the role of various ice dynamic and climatic processes controlling the evolution of ice sheets, coupled ice-sheet–bedrock modelling can also help to remove the isostatic component from surface measurements to obtain ice-thickness changes, which are the relevant quantity for sea-level variations. The quality of such a calculation will depend on how well the model deals with ice and bedrock dynamics and on how well past mass balance changes can be described.

When studying the response of the Antarctic ice sheet to future climate changes, a further distinction needs to be made between surface mass balance changes and the dynamic response of the ice sheet. That is because a changing mass balance will significantly affect the distribution of ice thickness and surface slope. The resulting changes in driving stress will influence the ice flow, and thus the shape of the ice sheet, and this can in turn be expected to feed back on the mass balance components. In Antarctica, there is the additional effect of changes in ice discharge from the grounded ice sheet into the ice shelves, and the possibility of grounding-line migration. The latter directly affects the volume of grounded ice above floating, which is the relevant quantity controlling changes of ocean mass and sea level. A related aspect is the potential occurrence of unstable behaviour in the West Antarctic ice sheet (WAIS), with its buttressing ice shelves and bed so far below sea level, and proven record of ice-stream variability.

These issues are addressed here from a modelling perspective. The chapter is composed of two main parts plus a concluding summary. The first part discusses the type of models used to examine the Antarctic ice sheet. The emphasis is on three-dimensional whole ice-sheet studies as these are the tools required to investigate the overall response. It is discussed how such models are constructed and what they have taught us about the behaviour of the ice sheet. This is demonstrated with model results dealing with the current and future evolution of the Antarctic ice sheet. Much of this discussion relies on the author's own work, as to date only a few studies with complete ice-sheet/ice-shelf/lithosphere models have been published. In the second part of this chapter, the scope widens to process model studies and the potential for internally generated instability by both thermal and grounding-line mechanisms. The concluding section summarizes current knowledge gained from numerical models regarding the possible evolution of the Antarctic ice sheet during the third millennium.

13.2 Models of the Antarctic ice sheet

13.2.1 Types of models

Different types of numerical models have been applied to investigate the Antarctic ice sheet. A distinction is usually made between how these models embody horizontal space: either they study the dynamics of selected one-dimensional flow lines within the ice sheet, or they study the ice sheet in the full two-dimensional horizontal plane. The former type is often referred to as a flow-line or flow-band model and the latter as a planform model (cf. Hulbe and Payne (2001) for a recent review). Flow-line models have the principal advantage that they vastly reduce the amount of computation. Such models are therefore often used in an exploratory fashion to study the effects of a particular physical process or for situations where flow lines are strongly constrained laterally (e.g. for outlet glaciers). The disadvantage of flow-band models is that they cannot deal with situations in which the direction of ice flow or the geometry of a catchment area exhibits spatial and temporal variability. In those cases, planform modelling is more appropriate.

Planform models are distinguished by the way in which they incorporate the vertical dimension. Either processes operating through the vertical extent are incorporated by the use of a vertical average (e.g. isothermal models), in which case these models are referred to as two-dimensional planform or vertically integrated models; or these models incorporate vertical processes explicitly. Examples of such vertical processes include ice temperature, stress and velocity components, as well as fabric and water content. This class of models has been termed three-dimensional, quasi-three-dimensional, or 2.5-dimensional.

The latter terms emphasize the fact that a different set of assumptions is often employed in the vertical compared with the horizontal dimensions (a consequence of the great difference between the horizontal and vertical length-scales associated with ice masses). Very few models which incorporate the vertical dimension are truly three-dimensional because in most cases the direction of flow is determined by the surface gradient and does not vary with depth. For simplicity, however, we will continue to refer to planform models that resolve the vertical direction as three-dimensional.

Planform time-dependent modelling of ice sheets largely stems from early work by Jenssen (1977) and Mahaffy (1976). These papers develop work by Nye (1957) on what has become known as the shallow-ice approximation (Hutter, 1983). This approximation recognizes the disparity between the vertical and horizontal length-scales of ice flow, and implies flow by simple shear. This means that the gravitational driving stress is balanced by shear stresses, and that longitudinal strain rate components are neglected. The assumption requires slopes to be smoothed over a distance an order of magnitude greater than ice thickness to circumvent problems associated with small-scale bedrock irregularities. Although the assumption breaks down at the margin (large longitudinal and transverse flow gradients) and at the centre (longitudinal stress gradients prevailing), it has shown general applicability in large-scale ice-sheet modelling. The model by Mahaffy (1976) was vertically integrated and was developed as a computer program to find the heights of an arbitrary ice sheet on a rectangular grid. It incorporated Glen's flow law (Glen, 1955) for ice deformation by dislocation creep. However, in polar ice sheets the flow is also, to a large extent, a temperature-dependent problem. The first model that dealt with the flow–temperature coupling in a truly dynamic fashion was developed by Jenssen (1977). Jenssen introduced a scaled vertical co-ordinate, transformed the relevant continuity and thermodynamic equations, and presented a framework to solve the system numerically.

13.2.2 Vertically integrated whole ice-sheet models

Historically, large-scale modelling of the Antarctic ice sheet was pioneered by Budd, Jenssen and Radok (1971) in their *Derived Physical Characteristics of the Antarctic Ice Sheet*. This landmark work introduced many concepts and techniques that are still used in glaciology today. The study assumed that the ice sheet is in steady-state and that the ice always flows downhill (a consequence of the shallow-ice approximation). This allowed the identification of flow lines, along which a moving-column model was advected downstream to investigate two-dimensional (vertical-plane) temperature and velocity distributions. The flux of ice was determined by integrating snow accumulation along these flow lines. This so-called balance flux can then be used to determine the vertically averaged horizontal ice velocity, if ice thickness is known. A host of additional characteristics were also calculated by Budd *et al.* (1971), including ice residence times, vertical and horizontal strain rates, and gravitational stresses.

The first time-dependent ice-dynamical models of the Antarctic ice sheet were published in the early 1980s by Oerlemans (1982a,b) and Budd and Smith (1982). They were vertically integrated planform models coupled with simplified models of isostasy and ice-shelf formation. Both models had a coarse 100 km grid, considered isothermal ice deformation and did not accommodate for any special treatment of the flow between grounded and floating ice. Nevertheless, these models were able to reproduce the present-day distribution of ice thickness reasonably well. When submitted to sea-level and air-temperature forcing, these models highlighted the sensitivity of, in particular, the West Antarctic ice sheet.

13.2.3 Three-dimensional models

An important step forward in the numerical modelling of the Antarctic ice sheet came with the development of models which coupled the temporal evolution of ice flow and temperature (Herterich, 1988; Huybrechts and Oerlemans, 1988), and from models which considered ice flow, not only in the grounded ice sheet, but also across the grounding line and in the ice shelf (Böhmer and Herterich, 1990; Budd *et al.*, 1994; Huybrechts, 1990a,b; Huybrechts, 1992; Huybrechts and Oerlemans, 1990). This allowed one to address the evolution of the Antarctic ice sheet, without recourse to overly restrictive assumptions, and it set the stage for many of the large-scale modelling studies of the late 1980s and 1990s. Figure 13.1 shows the structure of one such model as it was described in Huybrechts (1992) and further refined in Huybrechts and De Wolde (1999) and Huybrechts (2002). The core of this model is a set of thermomechanically coupled equations for ice flow that are solved in three subdomains, namely the grounded ice sheet, the floating ice shelf and a stress transition zone in between, at the grounding line. This involves the simultaneous solution of conservation laws for momentum, mass and heat, supplemented with Glen's flow law. The flow within the three subdomains is coupled through a continuity equation for ice thickness, from which the temporal evolution of ice-sheet elevation and ice-sheet extent can be calculated. The latter is done by applying a flotation criterion, meaning that the ice-sheet geometry is entirely internally generated. The model equations are solved on a numerical grid resting on realistic bedrock topography with the finite-difference method.

In the Huybrechts model, grounded ice flow is assumed to result both from internal deformation and from basal sliding over the bed in those areas where the basal temperature is at the pressure melting point. Ice deformation in the ice-sheet domain results from shearing in horizontal planes. For the sliding velocity, a generalized Weertman relation is adopted, taking into account the effect of the subglacial water pressure. Ice shelves are included by iteratively solving a coupled set of equations for ice-shelf spreading, including the effect of lateral shearing induced by side walls and ice rises. At the grounding line, longitudinal stresses are taken into account in the effective stress term of the flow law. These additional stress terms are found by iteratively solving three coupled equations for depth-averaged horizontal stress deviators. Adjustment of the bed to changes of the ice load is taken into account by a variety of models (Le Meur and Huybrechts, 1996). In more recent versions, the bedrock model consists of a rigid elastic plate (lithosphere) that overlies a viscous asthenosphere. This means that the isostatic compensation not only considers the local load, but integrates the contributions from more remote locations, giving rise to deviations from local isostasy. For an appropriate choice of the viscous relaxation time,

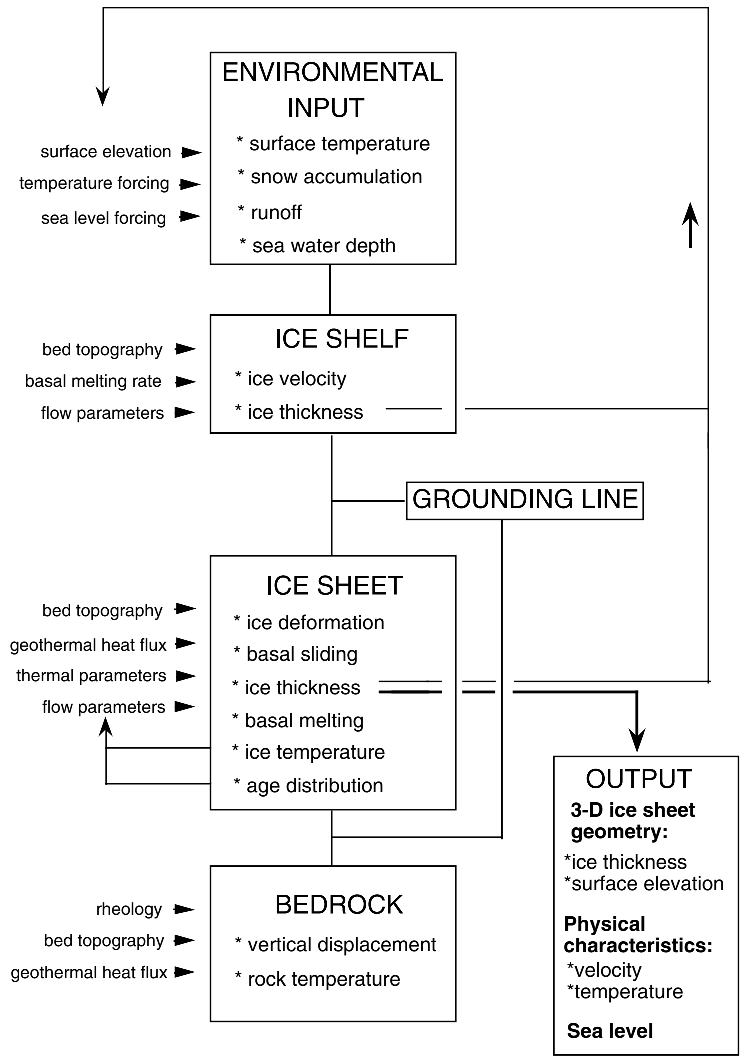

Figure 13.1. Structure of a three-dimensional ice-sheet model applied to the Antarctic ice sheet. The inputs are given at the left hand side. The model is driven by prescribed changes in environmental boundary conditions and has ice shelves, grounded ice and bed adjustment as major components. The position of the grounding line is not prescribed, but internally generated. Ice thickness feeds back on surface elevation, an important parameter for the calculation of the mass balance. The model essentially outputs the time-dependent ice-sheet geometry and the coupled temperature and velocity fields. (After Huybrechts (1992).)

this treatment produces results close to those from a sophisticated self-gravitating spherical visco-elastic Earth model, while at the same time being much more efficient in terms of computational overhead (Le Meur and Huybrechts, 1996). The horizontal grid resolution of the model is 40 km, and it has 11 layers in the vertical. This resolution was increased in recent work (Huybrechts, 2002) to 20 km and 31 layers, respectively, in an effort to better model concentrated flow in ice streams and outlet glaciers. This at the same time allowed the use of upgraded data sets for bedrock elevation and precipitation rate (Huybrechts *et al.*, 2000). Including a calculation of heat conduction in the bedrock, this gives rise to about 3×10^6 grid nodes. The numerical model used by Budd *et al.* (1994) and Warner and Budd (1998) also has a 20 km spatial resolution and was developed along similar lines, but with a somewhat simpler treatment of ice-shelf flow and no transition zone at the grounding line.

Interaction with the atmosphere and the ocean is, in these models, effectuated by prescribing the climatic forcing, consisting of the surface mass balance (accumulation minus ablation), surface temperature and the basal melting rate below the ice shelves. Changes in these parameters are usually parameterized in terms of air temperature. Precipitation rates are based on its present distribution and are perturbed in different climates according to sensitivities derived from ice cores or climate models. Melt-water runoff, if any, is obtained from the positive degree-day method (Braithwaite and Olesen, 1989; Reeh, 1991). This type of model is usually driven by time series of regional temperature changes (available from ice-core studies) and by the eustatic component of sea-level change. They have been used to address two main issues: the expansion and contraction of the Antarctic ice sheet during the glacial–inter-glacial cycles and the likely effects of greenhouse-induced polar warming. The answers to these questions rely on the complex interactions between grounding-line migration, interior ice-thickness changes and varying accumulation rates, as well as isostatic response, basal melting rate, ice temperature and ice viscosity.

Recent model developments have concentrated on ways of incorporating the dynamics of key areas that occur at the sub-grid scale and are therefore not well represented on 20 to 40 km grids. These principally concern the grounding line and areas of concentrated flow in outlet glaciers and ice streams, as well as details of the flow at ice divides. Such areas are characterized by large stress gradients, in which the approximations made in shallow ice-sheet models are known to break down. Ritz, Rommelaere and Dumas (2001) introduced the concept of a ‘dragging ice shelf’ to incorporate ice-stream dynamics, which is particularly important for the Siple Coast area of the West Antarctic ice sheet. In their model, inland ice is differentiated from an ice-stream zone by the magnitude of basal drag. This is based on the observation that ice-stream zones are characterized by low surface slopes, and thus low driving stresses, but have fast sliding. Ritz *et al.* (2002) treat these zones as semi-grounded ice shelves, and replace the shallow-ice approximation by a set of equations for ice-shelf flow to which basal drag is added. Gross model behaviour turns out to be quite similar to the whole ice-sheet models of Huybrechts, except that, in the Ritz *et al.* (Grenoble) model, the West Antarctic ice sheet has a lower surface slope near the grounding line and the break in the slope occurs further upstream at the place where the dragging ice shelf joins the inland ice subject to the shallow-ice approximation. One consequence is that grounding-line retreat in the Grenoble model occurs more readily in response to rising sea levels (Huybrechts *et al.*, 1998). Another way of incorporating smaller-scale features is to nest detailed higher-order models at higher resolution within a whole ice-sheet model. First attempts in this direction for limited inland areas near ice divides were presented in Greve *et al.* (1999) and Savvin *et al.* (2000).

Three-dimensional models have also been applied to the WAIS separately. Payne (1999) used the shallow-ice approximation in a model with fixed grounding line to investigate the interaction between thermomechanical coupling and basal sliding to generate steady-state oscillations in the ice flow. Hulbe and MacAyeal (1999) developed a numerical model which stands apart from other models in that it uses finite elements rather than finite differences to discretize the equations of motion. Their dynamic/thermodynamic model couples inland ice flow with flow in ice streams and ice shelves. The finite-element method eliminates the need for special parameterizations at flow regime boundaries, so that the ice flows smoothly from one regime to another. The method also allows for variable model resolution so one can concentrate computational effort on features of particular interest. However, the changing spatial patterns inherent in systems evolving over time pose a challenge in the application of finite-element models. Adaptive mesh generation is the best solution to that challenge, but has rarely been used in glaciology. Another limitation of the finite-element method is the associated computational burden. Therefore, principally due to their ease of use, finite differences have proven the more popular technique for ice-sheet modelling, in particular when the interest is in whole ice-sheet behaviour over longer periods of time.

13.3 Modelling the response of the Antarctic ice sheet

13.3.1 Modelling the present ice-sheet evolution

Modelling the quaternary evolution of the Antarctic ice sheet and its underlying bed is a way to obtain an estimate of the present-day ice-dynamic evolution unaffected by recent twentieth-century mass balance effects. The simulation requires time-dependent boundary conditions over a period long enough for the model to forget its initial start-up conditions. Long integrations over the last glacial cycles were analysed in Budd, Coutts and Warner (1998), Huybrechts (2002), Huybrechts and De Wolde (1999), Huybrechts and Le Meur (1999), and Ritz *et al.* (2001). Figure 13.2 shows the evolution of key glaciological variables in a typical run with the Huybrechts model over the last two glacial cycles, with forcing derived from the Vostok ice core (Petit *et al.*, 1999) and the SPECMAP sea-level stack (Imbrie *et al.*, 1984). Changing ice volume is principally a consequence of the areal expansion and contraction of the grounded ice sheet. Regional changes in ice thickness arise from these fluctuations in the location of the grounded ice sheet, and are further modulated by changes in ice temperature (cooler temperatures during a glacial result in a higher viscosity and thicker ice) and accumulation rate (reduced in cooler climates).

In the Huybrechts models, ice-sheet expansion during a glacial period mainly occurs over the Ronne-Filchner, Ross and Amery basins, and along the Antarctic peninsula. Around the East Antarctic perimeter, grounding-line advance is limited and is constrained by the proximity of the present-day grounding line to the continental shelf edge. During the last glacial maximum (LGM), modelled surface elevations over most of West Antarctica and the

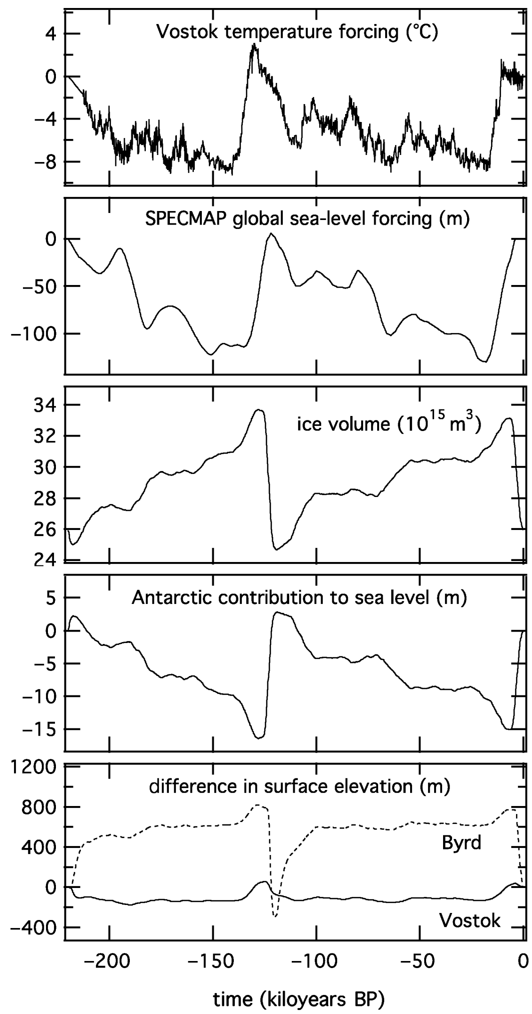

Figure 13.2. Forcing (mean annual air temperature and eustatic sea level) and predicted evolution of key glaciological variables (ice volume, contribution to sea level and local surface elevation changes at Byrd and Vostok stations) in a typical three-dimensional model experiment over the last two glacial cycles. Long spin-up times are required to model the current evolution of the ice sheet, which depends on its past history back to the last glacial period. (Based on the ice-sheet model described in Huybrechts (1992).)

Antarctic peninsula were up to 2000 m higher than present in direct response to grounding-line advance. Over central East Antarctica, surface elevations at the LGM were 100–200 m lower because of the lower accumulation rates. Holocene grounding-line retreat lags the eustatic forcing by some 10 000 years. This behaviour is related to the existence of thresholds for grounding-line retreat, and to the offsetting effect of late-glacial warming leading to enhanced accumulation rates and a thickening at the margin. In the model, most of the

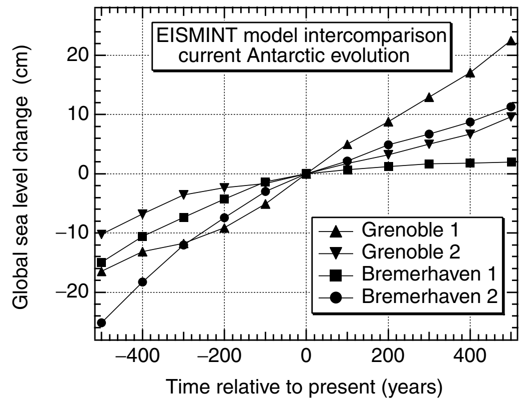

Figure 13.3. Modelled evolution of ice-sheet volume (represented as sea-level equivalent) centred at the present time resulting from on-going adjustments to climate change over the last glacial cycle. Data are from the Antarctic models that participated in the EISMINT intercomparison exercise. The Grenoble results were obtained with a model similar to the one in Ritz *et al.* (2001), and the Bremerhaven results are from the Huybrechts model. (From Huybrechts *et al.* (1998).)

retreat in West Antarctica occurs after 10 000 years BP. This timing is in line with recent geological evidence (Conway *et al.*, 1999; Ingolffson *et al.*, 1998) and is supported by some interpretations of relative sea-level data (Tushingham and Peltier, 1991). Nevertheless, small phase shifts in the input sea-level time series, inadequate representation of ice-stream dynamics and uncertainties in the Earth's rheological parameters may all have a significant effect on the model outcome.

Nonetheless, an important conclusion from this work is that the Antarctic ice sheet on the whole is still shrinking in response to grounding-line retreat during the Holocene. Experiments conducted as part of the European ice sheet modelling initiative (EISMINT) intercomparison exercise (Huybrechts *et al.*, 1998) confirm that the average Antarctic evolution at present is negative. Four different Antarctic models yield a sea-level contribution of between +0.1 and +0.5 mm per year averaged over the last 500 years (Figure 13.3), which corresponds to an average thinning of the ice sheet of between −0.3 and −1.6 cm of ice equivalent per year. Similar values of between +0.39 and +0.54 mm per year of sea-level change over the last 200 years have been found in Huybrechts and De Wolde (1999) and Huybrechts and Le Meur (1999). The geographical distribution of the local trend of Antarctic ice thickness, surface elevation and bed elevation in the latter study are displayed in Figure 13.4. The most pronounced feature is related to on-going grounding-line retreat along the Ross and Weddell Sea margins, which affects most of the West Antarctic ice sheet and nearby parts of the East Antarctic ice sheet. Local evolution rates range between −300 mm per year and +100 mm per year in West Antarctica, in contrast to the on-going slow thickening of several millimetres per year in interior East Antarctica. The latter is a direct consequence of the roughly doubled of accumulation rates following the last glacial–inter-glacial transition between 15 000 and 10 000 years BP.

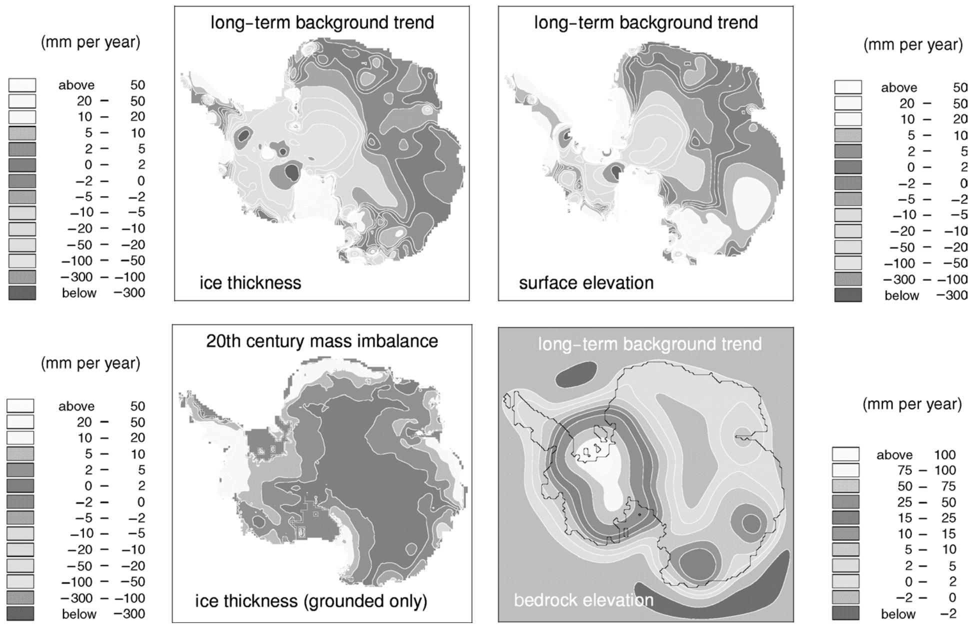

Figure 13.4. Long-term evolution patterns of ice thickness, surface elevation and bedrock elevation predicted by a coupled ice-sheet/visco-elastic bedrock model applied to the glacial cycles. The long-term patterns represent the mean evolution over the last 200 years and exclude the effects of twentieth-century climate change (Huybrechts and Le Meur, 1999). The lower left panel shows ice-thickness changes resulting from mass balance changes during the twentieth-century as predicted by scaling the ECHAM4/OPYC3 T106 patterns by the underlying ECHAM4 T42 resolution base trend (cf. Figures 13.6 and 13.7).

Glacio-isostatic modelling of the solid earth beneath the Antarctic ice sheet with prescribed ice-sheet evolution (James and Ivins, 1998) gives similar uplift rates to those presented in Figure 13.4, indicating that the underlying ice-sheet scenarios and bedrock models are similar, but observations are lacking to validate the generated uplift rates. By contrast, Budd *et al.* (1998) find that Antarctic ice volume is currently increasing at a rate of about 0.08 mm per year of sea-level lowering because in their modelling the Antarctic ice sheet was actually smaller during the LGM than it is today (for which there is, however, little independent evidence) and the effect of the higher accumulation rates during the Holocene dominates over the effects of grounding-line changes. Model simulations of this kind do not include the possible effects of changes in climate during the twentieth century. Simulations described further below, in which an ice-sheet model is integrated using changes in temperature and precipitation derived from AOGCM experiments, suggest that anthropogenic climate change could have produced an additional contribution of between –0.2 to 0.0 mm per year of sea level from increased accumulation in Antarctica over the last 100 years. Figure 13.4 shows a typical example of the corresponding distribution of ice-thickness changes, which are generally positive and of the order of 0–2 cm per year.

13.3.2 Modelling the mass balance and its changes

Fundamental to the understanding of the response of the Antarctic ice sheet to climate change is its surface mass balance. Except in local regions near the coast, and in isolated blue-ice areas with their own micro-climate, surface ablation is at present negligible on the Antarctic ice sheet. That is because of the very low air temperatures, which remain well below freezing throughout the year, even in the summer at low elevation. The only exception is the northern tip of the Antarctic peninsula, but this part is characterized by steep local glaciers and covers only a very small fraction of the total Antarctic area. Thus, the primary mechanism by which climate changes affect the evolution of the Antarctic ice sheet is surface accumulation. Any deviation from its longer-term average will have an immediate effect on the total ice volume. The effect is dominant on decadal to century timescales, because ice discharge changes over longer periods. Climate changes also affect the basal melting rate below the ice shelves, but these can only influence the grounded ice sheet indirectly through weakening of the ice shelves, causing possible grounding-line retreat or enhanced outflow. Other mass balance components include blowing snow, sublimation and basal melting below the grounded ice sheet, but these components are either very small or are already accounted for in surface accumulation estimates.

Several approaches have been taken to estimate the annual snow-fall distribution over the ice sheet, ranging from classical compilations of *in situ* observations (Giovinetto and Zwally, 2000; Huybrechts *et al.*, 2000; Vaughan *et al.*, 1999), to atmospheric moisture convergence analysis based on meteorological data (Bromwich, Cullather and Van Woert, 1998; Budd, Reid and Minty, 1995; Turner *et al.*, 1999), remotely sensed brightness temperatures of dry snow (Giovinetto and Zwally, 1995a) and studies with general circulation models (Krinner *et al.*, 1997). Recent accumulation estimates over all the ice sheet (including the ice shelves) display a tendency for convergence within a range of $2100–2400 \times 10^{12}$ kg per year (Church *et al.*, 2001), suggesting a remaining error of perhaps less than 10%. Nevertheless, the coverage of *in situ* data for validation of model results is still very poor over large areas, especially over the East Antarctic plateau.

To first order, it can be assumed that changes in surface accumulation are related to air temperature, because this controls the amount of water vapour that can be advected inland. Robin (1977) and Lorius *et al.* (1985) have suggested that accumulation is proportional to the saturated water vapour pressure of the air circulating above the surface inversion layer. Such a relationship appears to be particularly strong over the inland plateau and has been verified on the glacial–inter-glacial timescale from the ^{10}Be content in the Vostok ice core (Yiou *et al.*, 1985). Together with a degree-day model for melt-water runoff, this relation allows us to gain useful insight into the dependence of the surface mass balance on temperature change (Figure 13.5). As expected, accumulation increases steadily with temperature and runoff is essentially zero for present conditions. The total surface mass balance increases by 5.9% for a uniform temperature rise of 1 °C. The combined effect of changes in accumulation and melt-water runoff is an increase in net surface balance for a warming up to 5.3 °C; only temperature increases in excess of 8.3 °C would lead to

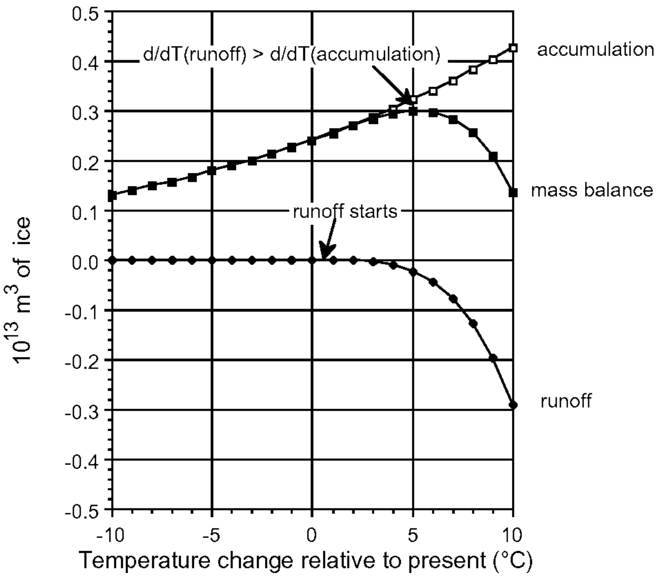

Figure 13.5. Dependence of Antarctic mass balance components on temperature relative to present. Runoff is calculated with a degree-day model, and accumulation changes are derived proportional to the saturated vapour pressure at the temperature above the surface inversion layer. For the temperature range relevant to future greenhouse warming, the relation between surface mass balance and mean air temperature is double-valued. (From Huybrechts and Oerlemans (1990).)

a reduced surface mass balance, any smaller perturbation leading to a net mass balance increase (Huybrechts and Oerlemans, 1990). These numbers are similar to those obtained by Fastook and Prentice (1994), underlining how a moderately warmer climate is likely to lead to Antarctic ice-sheet growth, and, hence, a sea-level fall from this source.

More sophisticated sensitivity analyses of Antarctica's surface mass balance have used multiple regression analyses (Fortuin and Oerlemans, 1990; Giovinetto and Zwally, 1995b), regional atmospheric models (Van Lipzig, 1999) and GCMs (Ohmura, Wild and Bengtsson, 1996; Smith, Budd and Reid, 1998; Thompson and Pollard, 1997; Wild and Ohmura, 2000). Recent progress has particularly been made with several coupled AOGCMs, especially in the 'time-slice' mode in which a high resolution model is driven by output from a low resolution transient experiment for a limited duration of time. Model resolution of typically 100 km allows for a more realistic topography crucial to resolve temperature gradients and orographic forcing of precipitation better along the steep margin of the Antarctic ice sheet. Figure 13.6 shows predicted patterns of climate change from the time-slice experiments conducted with the ECHAM4 model at T106 resolution (Wild and Ohmura, 2000). Both the simulated temperature and precipitation patterns for the present climate are generally close to compilations based on observations. In line with theoretical arguments, the model generally predicts increases of precipitation and temperature for doubled atmospheric CO_2

conditions, except over parts of Wilkes Land, for which there is no clear explanation. It can also be inferred from the summer temperature plots shown in Figure 13.6 that even in the $2 \times CO_2$ scenario, melting remains insignificant as the 0 °C isoline does not reach the Antarctic coast.

Table 13.2 summarizes the mass balance sensitivity for a 1 °C temperature rise from recent climate studies, expressed in equivalent global sea-level change. All data yield a precipitation increase corresponding to a fall of sea level of some 0.5 mm/°C per year. The sensitivity for the case that the change in accumulation is set proportional to the relative change in saturation vapour pressure is at the lower end of the sensitivity range, suggesting that in a warmer climate changes in atmospheric circulation and increased moisture advection can become equally important to enhance accumulation, in particular close to the ice-sheet margin (Bromwich, 1995; Steig, 1997; Van Lipzig, 1999). Both ECHAM3 and ECHAM4/OPYC3 give a similar specific balance change over the ice sheet for doubled versus present atmospheric CO_2 to that found by Thompson and Pollard (1997).

Very little is known about basal melting rates below the ice shelves and on how or how fast these could respond to climatic changes. Large-scale estimates put the total melting below ice shelves at between 320×10^{12} kg per year (Kotlyakov, Losev and Loseva, 1978) and 756×10^{12} kg per year (Jacobs, Hellmer and Jenkins, 1996), or an average rate of between 0.25 and 0.55 m per year. However, the data, limited as they are, point to a large spatial variation: melt rates have been inferred in excess of 10 m per year below Pine Island glacier (Jenkins *et al.*, 1997), whereas evidence has been found for large areas of basal accretion below the Filchner-Ronne ice shelf (Oerter *et al.*, 1992). Available oceanographic studies do not permit the establishment of a clear relation between climate change, oceanic circulation, oceanic temperature and basal melting or freezing rates (Nicholls, 1997; Williams, Jenkins and Determann 1998). Factors such as summer ocean warming, length of period with open water, thermohaline properties of the source water and the details of the water circulation below the ice shelves have all been mentioned to play a role. A model study by Williams, Warner and Budd (1998) shows a quadrupling of the basal melt rate below the Amery ice shelf for an adjacent sea warming of 1 °C, but another study by Nicholls (1997) claims that climatic warming would reduce melting rates below the Ronne-Filchner ice shelf through alteration to sea-ice formation and the thermohaline circulation. Larger melt rates would thin the ice shelves, but additional feedback loops may be involved. For instance, selective removal of the warmest and thus softest ice from the ice column would decrease the depth-averaged ice temperature and thus stiffen the remaining ice, thereby decreasing ice-shelf strain rates (MacAyeal and Thomas, 1986). Many of these routes to ice-shelf changes are not represented in current ice-sheet models. Nevertheless, when rapid thinning occurs close to the grounding line, grounding-line retreat can be induced. In large-scale ice-sheet models, this occurs in two ways: steeper gradients across the grounding zone cause larger driving stresses and higher deviatoric stress gradients across the grounding zone which lead to increased strain rates and, hence, a speed-up of the grounded ice and subsequent thinning (Huybrechts and De Wolde, 1999; Warner and Budd, 1998).

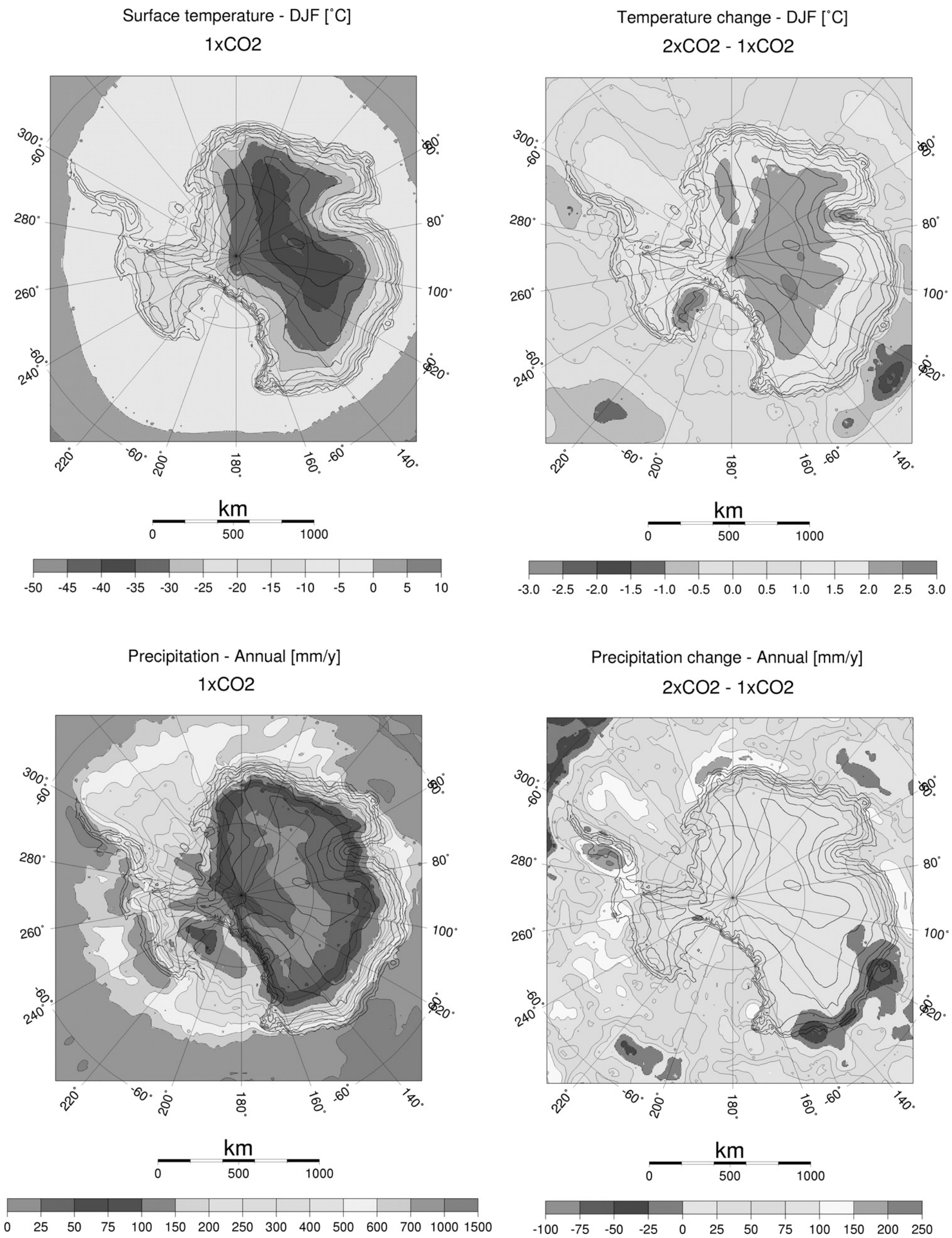

Figure 13.6. Patterns of climatic change over the Antarctic ice sheet from high resolution time-slice experiments with the ECHAM4/OPYC3 T106 model. Shown are the modelled mean annual precipitation and summer surface temperature (DJF) for the present climate (decade 1971–1980) as well as their changes at the time of doubled atmospheric CO_2 (decade 2041–2050). Over the ice sheet, total precipitation (expressed in water equivalent) exceeds total accumulation by the fractions of evaporation and liquid precipitation, but these latter components are usually small (the sublimation) or negligible (the rain fraction). (Redrawn from the information published in Wild and Ohmura (2000).)

Table 13.2. *Sensitivity of Antarctic mass balance to a 1 °C climatic warming.*

dB/dT is the mass balance sensitivity to local surface temperature change expressed as sea-level equivalent.

Source	dB/dT (mm °C per year)	Method
Huybrechts and Oerlemans (1990)	−0.36	change in accumulation proportional to saturation vapour pressure
Giovinetto and Zwally (1995b)	−0.80[a]	multiple regression of accumulation to sea-ice extent and temperature
Ohmura *et al.* (1996)	−0.41[b]	ECHAM3/T106 time-slice ($2 \times CO_2 - 1 \times CO_2$)
Smith *et al.* (1998)	−0.40	CSIRO9/T63 GCM forced with SSTs 1950–99
Van Lipzig (1999)	−0.47[c]	regional atmospheric climate model forced with ECMWF re-analysis data
Wild and Ohmura (2000)	−0.48[b]	ECHAM4/OPYC3/T106 time slice ($2 \times CO_2 - 1 \times CO_2$)

[a] Assuming sea-ice extent decrease of 150 km/°C.
[b] Estimated from published data and the original time-slice results.
[c] Derived from an experiment with +2 °C external temperature forcing.

13.3.3 Modelling the future evolution of the Antarctic ice sheet

Response during the twenty-first century

Three-dimensional modelling studies all indicate that the dynamic response of the Antarctic ice sheet can be neglected on a century timescale, except when melting rates below the ice shelves are prescribed to rise by in excess of 1 m per year (Budd *et al.*, 1994; Huybrechts and De Wolde, 1999; Huybrechts and Oerlemans, 1990; O'Farrell *et al.*, 1997; Warner and Budd, 1998). This means that the response is essentially static, and thus that the ice flow on this timescale hardly reacts to changes in surface mass balance. Depending on the warming scenario and on how accumulation rates change with temperature, increased ice volume during the twenty-first century typically leads to global sea-level drops of the order of some 10 cm. For instance, when forced with temperature changes from a two-dimensional climate and ocean model forced by greenhouse gas rises of, respectively, 0.5% (low scenario), 1% (mid scenario) and 1.5% (high scenario) per year, Huybrechts and

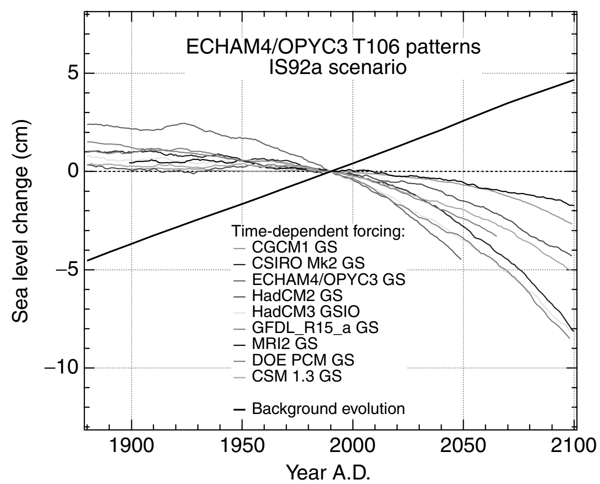

Figure 13.7. Volume changes of the Antarctic ice sheet during the twentieth and twenty-first centuries using temperature and precipitation changes from AOGCM experiments following the IS92a scenario, including the direct effect of sulphate aerosols, to derive boundary conditions for a three-dimensional ice-sheet model. The acronyms refer to the AOGCMs providing the base trend used to scale the ECHAM4/OPYC3 T106 climate-change patterns, cf. Figure 13.6. The thick black line shows the modelled long-term background trend. These experiments served as a base for the sea-level predictions of the IPCC Third Assessment Report (Church *et al.*, 2001).

De Wolde (1999) find Antarctic sea-level contributions between 1990 and 2100 to vary between −4.3 and −11.4 cm. In these runs, there is minimal surface melting, reaching only 5% of the total accumulation by AD 2100 in the high scenario. These numbers exclude the long-term background term, which was found in this study to be +5.0 cm during the same time interval. Similar changes during the twenty-first century due to accumulation changes are reported in Budd *et al.* (1994), O'Farrell *et al.* (1997) and Warner and Budd (1998).

The ice-sheet model of Huybrechts and De Wolde (1999) was used to make projections of Antarctic ice-sheet mass changes for the IPCC Third Assessment Report (Church *et al.*, 2001). Boundary conditions of temperature and precipitation were derived by perturbing present-day climatologies according to the geographically and spatially dependent patterns predicted by the T106 ECHAM4 model (Wild and Ohmura, 2000) for a doubling of CO_2 under the IS92a scenario. To generate time-dependent boundary conditions, these patterns were scaled with the area-average changes over the ice sheets as a function of time for available AOGCM results. The result is shown in Figure 13.7, suggesting global sea-level changes during the twenty-first century between –2 and –8 cm. These results were subsequently regressed against global mean temperature to enable further scaling to take into account the complete range of IPCC temperature predictions for the most recent SRES emission scenarios. Taking into account the background evolution and various sources of uncertainties, it yielded a predicted Antarctic contribution to global sea-level change between 1990 and 2100 of between −19 and +5 cm, which range can be considered as a 95% confidence interval (Church *et al.*, 2001).

Response during the third millennium and beyond

On centennial to millennial timescales, predictions should be based on dynamic rather than static simulations. The experiments discussed in Huybrechts and Oerlemans (1990) and Huybrechts and De Wolde (1999) demonstrate that ice dynamics counteract the accumulation-only response. Several mechanisms can be distinguished depending on the strength of the warming. For warmings below about 5 °C, runoff remains insignificant and there is hardly any change in the position of the grounding line. Under these circumstances, the inclusion of ice dynamics causes less growth, mainly because of an increase of the ice flux across the grounding line, which in part counteracts the thickening effect due to the increased accumulation rates. This increase of the flux across the grounding line is a result of both the local thickening at low elevations near to the grounding line, producing higher shear stresses and thus higher velocities, and of an increased ice-mass discharge on the ice shelves that pulls the ice out of the grounded ice sheet and is effective some distance inland. The effect becomes progressively stronger in time, and counteracts the static effect by 6% after 100 years, 30% after 500 years and more than 50% after 1000 years, reflecting response timescales at the margin (Huybrechts and De Wolde, 1999). According to this model study, the dynamic response leads to a sea-level change (relative to the background effect) of −50 cm for a surface warming of +3 °C and of −80 cm for a warming of +5.5 °C by the end of the third millennium (Figure 13.8).

For warmings that exceed 5–6 °C, on the other hand, significant grounding-line retreat sets in. In this model experiment, that is largely due to increased surface melting around the ice-sheet edge, leading to a thinning of grounded ice, but also to the effect of a warming ice shelf. The latter causes the ice shelf to deform more easily, which leads to larger flow velocities and equally a thinning. In Huybrechts and De Wolde (1999), the result is a sea-level rise of as much as +80 cm after 1000 years of simulated time for a warming of 8 °C. Grounding-line retreat is centred along the Antarctic peninsula and the northern-most parts of the East Antarctic perimeter (Wilkes Land in particular), with little change predicted to occur around the Ronne-Filchner and Ross ice shelves, because, even with a warming of 8 °C, no runoff takes place at their grounding zones. This response is apart from any changes in the oceanic circulation that may affect basal melting rates.

In the model studies performed by the Australian group (Budd *et al.*, 1994; O'Farrell *et al.*, 1997; Warner and Budd, 1998), large increases in bottom melting are the dominant factor in the longer-term response of the Antarctic ice sheet, even for moderate climate warmings of a few degrees. Budd *et al.* (1994) found that, without increased accumulation, the increased basal melt of 10 m per year would greatly reduce ice shelves and contribute to a sea-level rise of over 0.6 m after 500 years, but no drastic retreat of the grounding line. Reduction of grounded ice volume was mainly by high strain rates and thinning rates

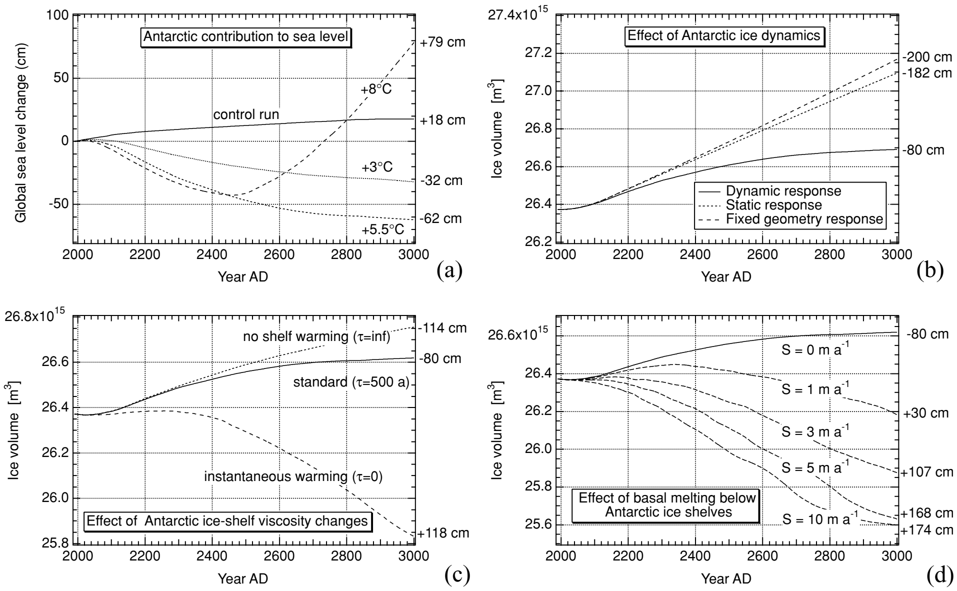

Figure 13.8. Modelled response of the Antarctic ice sheet during the third millennium subject to climate-warming scenarios predicted by a two-dimensional climate and ocean model forced by greenhouse gas concentration rises between two and eight times the present CO_2 by AD 2130, and kept constant after that (a). Also shown are the effects of ice dynamics (b) and the role of viscosity changes (c) and basal melting rates below the ice shelves (d) for the middle scenario ($4 \times CO_2$, $+5.5\,^\circ$C). S is the applied basal melting rate and τ is the characteristic e-folding timescale for temperature adjustment in the ice shelf. The numbers in the right hand margins refer to the equivalent sea-level change between AD 1990 and 3000. (From Huybrechts and De Wolde (1999).)

near grounding lines. With a similar ice-sheet model, but different forcing derived from simulations with the CSIRO9 Mk1 fully coupled AOGCM, O'Farrell *et al.* (1997) find a sea-level rise of 0.21 m after 500 years for a transient experiment with basal melt rates evolving up to 18.6 m per year. In the study by Warner and Budd (1998), a bottom melt rate of 5 m per year causes the demise of WAIS ice shelves in a few hundred years and would remove the marine portions of the West Antarctic ice sheet and a retreat of coastal ice toward more firmly grounded regions elsewhere over a time period of about 1000 years. Predicted rates of sea-level rise are, in this study, up to between 1.5 and 3.0 mm per year depending on whether accumulation rates increase together with the warming. Similar volume reductions under conditions of high bottom melting were obtained in Huybrechts and De Wolde (1999). Allowing for runoff in addition to increased accumulation, they found a sea-level rise of more than 150 cm after 1000 years for a scenario involving a warming of $+5.5\,^\circ$C and a basal melting rate of 5 m per year (Figure 13.8). Though these are large shrinking rates, obtained under severe conditions of climate change, they cannot be considered to support the concept of a catastrophic collapse or strongly unstable behaviour of the WAIS, which

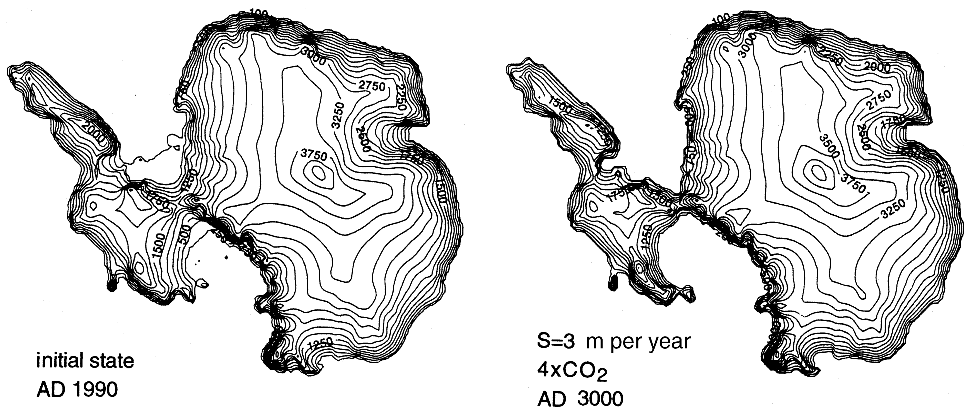

Figure 13.9. Snapshots of Antarctic surface elevation evolution for an experiment in which $4 \times CO_2$ and a uniform melting rate of 3 m per year below the ice shelves are applied. The plots demonstrate that changes are mainly restricted to grounding-line recession in West Antarctica, whereas the East Antarctic ice sheet thickens slightly. The righthand plot corresponds to a global sea-level rise of 1.07 m with respect to the 1990 initial state. Contour interval is 250 m; the lowest contour is close to the grounding line. (After Huybrechts and De Wolde (1999).)

is usually defined to mean its demise within several centuries, implying sea-level rises in excess of 10 mm per year (Mercer, 1978; Oppenheimer, 1998; Vaughan and Spouge, 2002).

The study by Huybrechts and De Wolde (1999) also highlighted the effect of reduced ice-shelf viscosity following a climatic warming to enhance strain rates and grounding-line retreat. The ice-sheet geometries shown in Figure 13.9 show that bottom melting causes most grounding-line retreat for the Ross ice shelf. Changes in the geometry of the East Antarctic ice sheet, on the other hand, are hardly distinguishable on the scale of the plots, but involve a thickening of up to 100 m on the plateau and some grounding-line retreat along the most over-deepened outlet glaciers in the lower right quadrant of the plotted ice sheet (in particular Totten, Ninis and Mertz glaciers). However, it should be stressed that the mechanics of grounding-line migration are not fully understood, and so these model results depend on how ice-sheet flow is coupled with ice-shelf flow. Also, none of these three-dimensional models adequately includes ice streams, and it may be that the ice streams are instrumental in controlling the behaviour and future evolution of the ice sheet in West Antarctica.

Independent of bottom melting below the ice shelves and ice-stream dynamics, surface melting sets an upper temperature limit on the viability of the Antarctic ice sheet, because runoff would eventually become the dominant wastage mechanism. For warmings of more than 10 °C, the mass balance at sea level around the Antarctic coast would be sufficiently negative so that the grounded ice sheet is no longer able to feed an ice-shelf. Also the WAIS ice shelves would disintegrate to near to their inland limits as summer temperatures rise above the thermal limit of ice-shelf viability believed to be responsible for the recent collapse of ice shelves at the northern tip of the Antarctic peninsula (Doake *et al.*, 1998;

Skvarca *et al.*, 1998; Vaughan and Doake, 1996). Disintegration of WAIS would, in that case, result because the WAIS cannot retreat to higher ground once its margins are subjected to surface melting and begin to recede (Huybrechts, 1994). Depending on the strength of the warming, such a disintegration would take at least a few millennia. Thresholds for disintegration of the East Antarctic ice sheet by surface melting involve warmings above 20 °C, a situation that has not occurred for at least the last 15 million years (Barker *et al.*, 1999), and which is far more than thought possible under any scenario of climatic change currently under consideration.

13.4 Potential sources of instability

The modelling studies reviewed above did not specifically address particular mechanisms that could cause runaway behaviour of the Antarctic ice sheet in the future. Such mechanisms have been identified in some glaciers, and a basic question is whether they could also occur in large ice sheets. Examples include the surging of valley glaciers and rapid retreat of tide-water glaciers. The search for instabilities in the Antarctic ice-sheet system has directed a great deal of research over the last two decades. Two mechanisms are potentially relevant for the Antarctic ice sheet. Grounding-line instability refers to an ice sheet based on a bed below sea level and is often mentioned in association with the West Antarctic ice sheet. Thermomechanically induced instabilities could occur in any type of grounded ice and have been discussed for both the East and West Antarctic ice sheets. An excellent review of potential sources of instability relevant to the West Antarctic ice sheet was given in Hulbe and Payne (2001), which is expanded upon in the discussion below.

13.4.1 Grounding-line instability

The marine nature of the West Antarctic ice sheet has led many glaciologists to believe that it may be inherently unstable and may respond drastically and irreversibly to a warming climate (Hughes, 1975; Lingle, 1984; Mercer, 1978; Thomas, 1979; Thomas and Bentley, 1978; Thomas, Sanderson and Rose, 1979; Weertman, 1974). This view is based on the idea that the large ice shelves surrounding West Antarctica control the discharge of inland ice and that a grounding-line tends to be unstable if the sea depth is greater than a critical depth and the sea floor slopes down toward the ice-sheet interior. The field evidence regarding WAIS (in-)stability is discussed in Chapter 12.

The original idea of the marine ice-sheet instability lies in the Weertman (1957) analysis of ice-shelf spreading. In that work, an expression was derived for unidirectional spreading of a confined ice shelf that leads to a creep thinning rate in the floating ice, and, thus, near the grounding line, which depends on the fourth power of ice thickness. Flow at the grounding line should therefore increase rapidly as a retreating grounding line progressively encounters deeper water and thus thicker ice, as is typically the case in West Antarctica.

This creates a positive feedback that will result in further retreat of the grounding line and a complete collapse, unless there is a sufficiently high bedrock sill on which the grounding line can achieve equilibrium (Thomas, 1979). The WAIS owes its existence, in an extension of that concept, to frictional drag from a combination of grounded ‘pinning points’ in the ice shelf and shear resistance along the sides of their enclosing embayments. That drag creates a ‘back-pressure’ from the ice shelves, which restrains the outflow of the grounded inland ice (Thomas *et al.*, 1979). If the effect of a warming were to weaken the ice shelves through increased calving and bottom melting, the ensuing reduction in back stress would strongly diminish their buttressing effect, and could initiate a collapse in as little as a century, causing sea level to rise at a mean rate of some 50 mm per year (Mercer, 1978).

However, the anchor upon which the instability hypothesis rests is the transmission of stress from floating to grounded ice. Van der Veen (1986) pointed out the inconsistency in incorporating ice-shelf dynamics at the grounding line, mainly because the ice-sheet/ice-shelf feedback is taken into account at one point only and a discontinuity is introduced when calculating creep thinning rates and ice velocities. In the Thomas *et al.* (1979) analysis, creep thinning at the grounding line is calculated from an expression based on ice-shelf spreading, but the movement at the grounding line is controlled by basal shear stresses. That excludes a negative feedback whereby increased advection from the ice sheet can compensate for the thinning. When calculating ice velocities explicitly along an entire flow line, Van der Veen (1985) found that the position of the grounding line was only little influenced by changes in ice-shelf back-pressure, and, furthermore, that an ice shelf is not needed to stop grounding-line retreat. In a further development, Hindmarsh (1993) argues that the transition zone between grounded ice and floating ice is of such limited extent that it is unlikely for any stress transmission to take place. Numerical simulations (Herterich, 1987; Lestringant, 1994) and an analytical analysis of stick-slip transitions (Barcilon and MacAyeal, 1993) support this view. Interpretation of strain rates across the grounding zone of Ekström ice shelf arrives at the same conclusion (Mayer and Huybrechts, 1999). Hence, the original marine ice-sheet instability hypothesis can be refuted on the grounds of the simplified model it was based on.

In reality, however, the situation of the WAIS is complicated by the presence of fast flowing ice streams whose characteristics blend gradually into those of the ice shelves and whose response times to changes at the grounding line appear to be very rapid (Alley and Whillans, 1991). There is a considerable body of evidence for ice-stream variability (Retzlaff and Bentley, 1993; Stephenson and Bindschadler, 1988), but the mechanisms for these oscillations are not well understood. They have been ascribed to processes such as basal water diversion to a neighbouring ice stream (Anandakrishnan and Alley, 1997) or thermomechanical interactions between competing catchment areas (Payne, 1999). Other mechanisms for dynamic changes have also been proposed at the interface of ice that is lubricated by sediment and water (Anandakrishnan *et al.*, 1998; Bell *et al.*, 1998; Blankenship *et al.*, 1986), the transverse shear zone where fast moving ice meets relatively static ice (Echelmeyer *et al.*, 1994; Jacobson and Raymond, 1998) and the ice-stream onset regions where slowly flowing inland ice accelerates into the ice streams. Just to what extent the ice streams contribute to the stability of the WAIS is in dispute. On the one hand, the inherent ability of ice streams to transport ice rapidly from the interior to the ocean indicates, in the view of some glaciologists, an enhanced capability for a drastically accelerated output flux. A contrary view is that the rapid response time of ice streams removes the flux imbalance at the grounding line that was the basis of the instability model, and that the purported grounding-line instability very well may not exist (Hindmarsh, 1993).

13.4.2 Thermomechanical instability

Three types of thermomechanical instability are identified in ice-sheet models. These relate to creep instability; the downstream transition from frozen to melting basal conditions; and the occurrence of warm-based ice encircled by frozen bed conditions. The first two processes are related to the temperature dependence of the flow law for ice. The third relies on the geometry of the basal temperature field. These instabilities are internal, in that they depend on flow-dependent thermal evolution of the ice sheet. Under suitable conditions, these instabilities can lead to free oscillations, i.e. cyclical behaviour without external forcing.

The temperature dependence of the flow properties of ice contains the potential for positive feedback between predicted ice velocity and temperature fields. As ice temperature increases, ice deformation rates will also increase. This leads to enhanced flow, increased dissipation and further warming. Clarke, Nitsan and Paterson (1977) introduced the term ‘creep instability’ for the process whereby an initial temperature anomaly leads to a runaway increase in ice velocity. Initial numerical studies of creep instability employed vertical, one-dimensional models of constant thickness resting on a bed with constant slope. In the study by Clarke *et al.* (1977), it was demonstrated that the feedback between temperature and ice flow can lead to bifurcation. Under specific conditions, multiple steady-states occur: either the ice sheet is thick and cold with low velocities, or it is thin and warm with high velocities, implying that the ice sheet can switch suddenly from the slow mode to the fast mode if external conditions change. Clarke *et al.* also carried out an analysis of linear stability, revealing that typical e-folding times for the runaway increase are in the 10^3 to 10^4 years range for large ice sheets. The study by Schubert and Yuen (1982) elaborates further on this problem. Their scenario calls on climatically enhanced accumulation to increase the thickness of the East Antarctic ice sheet above a critical value for the onset of an explosive shear heating instability. This would cause massive melting at the base and initiate a surge. In Yuen, Saari and Schubert (1986), conditions for the onset of this explosion were investigated. For certain values of ice rheological parameters, they found that a sudden increase in ice-sheet thickness by 1–2 km could lead to melting of the basal shear layer in only thousands of years.

One objection that can be made against these analyses, however, is that they are ‘local’: horizontal temperature advection and driving stress (ice-sheet geometry) are not allowed to react to the changing temperature and velocity fields. These processes would tend to dampen temperature perturbations. Numerical modelling studies conducted with a thermomechanical flow-band model suggest that, for the East Antarctic ice sheet, this horizontal heat advection prohibits the development of any runaway warming (Huybrechts and Oerlemans, 1988). Hulbe (1998) discusses the positive feedback in the context of a quasi-three-dimensional thermomechanical finite-element model that includes both horizontal and vertical temperature advection and diffusion. In that analysis, the tendency toward excessive heating in deep ice is mitigated firstly by corresponding large vertical strain rates, which thin the ice and thus increase upward diffusion of temperature, and secondly by enhanced downstream advection.

Payne (1995), Pattyn (1996) and Greve and MacAyeal (1996) studied a related form of thermomechanical instability. In these models, an instability arises because of an assumed abrupt increase in sliding velocity with the onset of basal melting. The sudden transition leads to a pronounced step in the ice-surface profile above the warm–cold ice transition. The steep surface slope in turn increases the gravitational driving stress and deformational velocity, and thus viscous dissipation also increases dramatically. Payne (1995) estimates a 16-fold increase in dissipation for a doubling of ice surface slope. The location of the warm–cold ice transition point can migrate rapidly upstream as a consequence of this localized heating and associated enhanced flow, causing a surge. Eventually, reduced ice thicknesses and enhanced cold-ice advection lead to stagnation. The extent of the bed area at the melting temperature depends on the air temperature and accumulation rate. For low temperatures, most of the glacier remains frozen to its bed, and oscillations are restricted to the downstream area. The period of the oscillations also changes as the accumulation rate changes: for low rates the dominant period is 6000–7000 years, for intermediate accumulation the period is 3000–4000 years, while for larger accumulation the oscillations become irregular with periods in excess of 10 000 years. Applying the same physics to the WAIS in three dimensions, Payne (1999) finds large spatial variability concentrated in the ice streams along the Siple Coast. This is attributed to competition between adjacent ice streams for increased drainage area, and leads to internally generated cycles of ice-stream growth and stagnation. Typical periods of these oscillations are between 5000 and 10 000 years. Despite large variability in output fluxes of individual ice streams by up to a factor of 5, the overall volume of the WAIS, however, hardly changed, supporting the suggestion that the ice streams may act to remove the imbalance of individual drainage basins (Hindmarsh, 1993).

A third form of internal instability is discussed by Oerlemans (1983) and MacAyeal (1992) in studies that seek cyclic behaviour in ice sheets. Both employ thermomechanical models that associate the presence of basal melt water with enhanced basal sliding. The latter model is applied to the WAIS only, uses ice-stream specific stress balance equations, but ignores horizontal temperature advection. Oerlemans (1983) uses a constant climate forcing, while MacAyeal (1992) specifies a climate cycle according to the Vostok ice-core record. The result in both cases is a cycle of slow ice-sheet growth and rapid discharge. Self-sustained oscillations were also found by Oerlemans and Van der Veen (1984) when employing a two-dimensional model of the Antarctic ice sheet. Their model contains a simplified calculation of the basal ice temperature as well as the feedback between ice flow and basal water. Also here, for certain values of the model parameters, ice thickness over

East Antarctica varied periodically with a time interval of typically a few thousand years. This was caused by the accumulation of basal water under the thicker parts. West Antarctica, on the other hand, remained close to a steady-state with extensive melting and high sliding velocities. Nevertheless, much also seems to depend on how sliding is parameterized in terms of basal water and on how effectively melt water is removed.

MacAyeal's (1992) inclusion of subglacial till dynamics leads to episodic ice-sheet fluctuations that are out of phase with the climate forcing. Collapses occur at times irregularly spaced relative to the ice-age cycle, because of the long time constant of the subglacial system. His modelling yielded three collapses in 10^6 years; it also produced iceberg pulses, with fluxes of 1000 to 3000 Gt per year (3–10 mm per year sea-level rise) lasting a few decades, about ten times during the million-year model run. These too occurred pseudo-randomly. The periodic behaviour relies on the development of basal ice at its melting point in the interior of the ice sheet, where ice is thick, while ice nearer the margins remains frozen to the bed. Eventually, the pool of warm-based ice breaks through the encircling cold-based ice, leading to a large, rapid surge. The thin, post-surge ice sheet refreezes to its bed, thickens over time and the cycle repeats. However, some approximations in MacAyeal's model remain crude, particularly those relating to the bed and to horizontal advection, which is not considered, thus precluding thermodynamic feedback with the flow from upstream. The spatial pattern of warm basal temperature in the interior and cold basal temperature near the margins is also the opposite to that predicted by the majority of WAIS thermomechanical models (Budd *et al.*, 1971; Huybrechts, 1992; Payne, 1999). The main process favouring warm-based divides is the increased thermal insulation afforded by thick ice. Processes favouring cold-based divides and warm-based margins are enhanced cold-ice advection at the divide and increased dissipation as ice discharge increases towards the margin. The models which predict cold-based interiors are physically more realistic because horizontal temperature advection is fully incorporated. Also, West Antarctic ice streams are already today known to be melting at their beds, yet none of them seem to be in a 'collapsing' mode.

13.5 Conclusions and further outlook

This chapter has discussed some of the models in use today to study the behaviour of the Antarctic ice sheet and their application in understanding the current and future response of the ice sheet, and has identified potential sources of internal instability. Although many aspects of large-scale ice-sheet models still remain rather crude and need to be developed further, a number of key results are starting to emerge. From the modelling evidence reviewed in this chapter, it appears that the Antarctic ice sheet is late in the cycle of ice-sheet growth and retreat and is still responding to the end of the last Ice Age. Most of the ongoing shrinking is concentrated in the West Antarctic ice sheet as a delayed response to post-glacial sea-level rise, whereas the East Antarctic ice sheet may be close to a stationary state, or growing slightly in response to the increased accumulation rates. Short and yet incomplete satellite altimetry records indeed support such a broad picture, although such a comparison should be reserved because of the different time periods involved (Wingham *et al.*, 1998). The background trend may well dominate the response of the ice sheet during the twenty-first century, and is probably of the order of an equivalent global sea-level rise of a few centimetres per century (Church *et al.*, 2001).

A climatic warming during the third millennium will probably lead to an increase in precipitation which has an immediate effect on Antarctic ice volume. Increased precipitation will increase the mass input onto the ice sheet, but any balancing increase in outflow will take centuries to millennia to compensate for the gain. So, in the short-term, the ice sheet will probably grow slowly relative to its current state of balance. There is, indeed, evidence, both from moisture flux modelling (Bromwich *et al.*, 1998) and glaciological observations (Mosley-Thompson *et al.*, 1999; Peel and Mulvaney, 1988), that that is already happening. As long as the warming remains moderate ($<5\,^{\circ}$C), surface melting is likely to remain insignificant for the mass balance, and the increase in accumulation will dominate over any increase in melting. Typical ice growth rates are predicted to be of the order of 5–10 cm per century of equivalent sea-level lowering. For larger warming, however, surface melting rates are predicted to become more important, especially at the Antarctic peninsula and around the East Antarctic perimeter. As a consequence, ice-sheet models exhibit thinning at the ice-sheet margin, and eventually grounding-line retreat results. It would take up to several centuries before these effects become apparent, but, depending on the strength of the warming, volume reductions equivalent to sea-level rises of several tens of centimetres per century are a distinct possibility (Huybrechts and De Wolde, 1999).

Faster changes could only arise from instability mechanisms able to increase the outflow by a multiple of the present amount. Thermomechanical instabilities have been identified in ice-sheet models, but may not be very realistic as most of these models address processes in isolation and often neglect crucial mechanisms shown to counteract any runaway behaviour. Also, these instability mechanisms operate over time periods of millennia and are therefore of less relevance on a human timescale. And, if they would occur, they are probably not linked to anthropogenic climatic warming, but to past climatic changes (MacAyeal, 1992). That is because ice sheets take thousands of years to respond to changes in surface temperature, as it takes that long for temperature changes to penetrate to the bed, and only there could increasing temperatures affect the flow rates. The grounding-line instability hypothesis as originally formulated in the 1970s is demonstrably a model artefact and can therefore not be invoked as a serious candidate to induce a collapse of the West Antarctic ice sheet. The dynamic connection between ice shelves and the Siple Coast ice streams may modify that view, but to date no credible model has been put to work that is able to produce an average speed-up of the total outflow by at least a factor of 10 required for a sea-level rise in excess of 10 mm per year (Bentley, 1997, 1998a,b).

On a multiple-century timescale, oceanic warming is another crucial factor that could affect the grounded ice sheet because of its potentially weakening effect on ice shelves. However, GCM studies have suggested that oceanic warming in the far Southern Ocean would be delayed by centuries compared with the rest of the world because of the large-scale sinking of surface waters around Antarctica (Manabe *et al.*, 1991). Recent spectacular breakups of the Larsen ice shelves in the Antarctic peninsula (Doake *et al.*, 1998) demonstrate the existence of an abrupt thermal limit on ice-shelf viability associated with regional atmospheric warming (Skvarca *et al.*, 1998). But the WAIS ice shelves are not immediately threatened by this mechanism, which would require a further warming of 10 °C before the −5 °C mean annual isotherm reached their ice fronts (Vaughan and Doake, 1996). Although atmospheric warming would increase the rate of deformation of the ice, causing the ice shelf to thin, response timescales are equally of the order of several hundred years (Rommelaere and MacAyeal, 1997). Furthermore, it can be questioned whether ice-shelf thinning would have any drastic effect on the inland ice. Models indicate that, in any case, very high average melting rates in excess of 10 m per year would be required to produce ice-sheet retreats equivalent to sea-level rises of the order of 1–3 mm per year (Huybrechts and De Wolde, 1999; Warner and Budd, 1998). Based on the current model evidence, it is therefore very unlikely that Antarctica would undergo major volume losses during the next few centuries, and perhaps even during most of the third millennium.

Even though much progress has been made in understanding and modelling of the Antarctic ice sheet, several lines of future investigation are apparent. These can be roughly divided into two groups: improved boundary and test data, and incorporation of more appropriate physics. Climate and mass balance related boundary conditions are obviously vital to simulate correctly ice-sheet evolution. Present-day atmospheric boundary conditions, such as mean annual air temperature and snow accumulation, are known to a level of accuracy commensurate with that required by ice-sheet models, but their patterns of change in past, as well as future, climates are poorly constrained. Palaeoclimatic data show that storm strengths and trajectories have changed in the past and have greatly affected accumulation, especially at the ice-sheet margin. This raises the possibility that future circulation changes will occur and will also affect precipitation. It is important that atmospheric and snow-surface processes be understood well enough so that model-based predictions of snow accumulation can be made directly, rather than using predictions of temperature and assumed temperature sensitivity or perturbation methods to estimate precipitation. Even more troublesome is the melt rate from the underside of the ice shelves, which may affect grounding lines but for which we have very limited data. The same is also true of the geothermal heat warming at the ice-sheet base, which exerts a crucial control on the spatial extent of basal melting, but for which there is virtually no data. In addition, the process of iceberg calving, and more generally the disintegration of ice shelves, is currently not well understood and therefore impossible to model with confidence.

Model validation and testing is important. A series of validation experiments were developed within the framework of the European ice sheet Modelling initiative (EISMINT) to provide tests for ice-shelf and ice-sheet numerics, thermomechanical coupling and planform models applied to the Antarctic and Greenland ice sheets (Huybrechts *et al.*, 1996, 1998; MacAyeal *et al.*, 1996; Payne *et al.*, 2000). These have enabled modellers to discover errors and numerical instabilities and to upgrade individual models. However, models should also be tested against field data, but this is hampered by a paucity of data at the appropriate spatial and temporal scales. In particular, models of basal thermal regime, basal hydrology and subglacial sediment deformation remain untested except at a very limited number of bore holes and by broad comparison with geophysical inferences from seismics or radio-echo sounding. A linked aspect concerns the incorporation of fast flowing outlet glaciers and ice streams, which are not described well in contemporary ice-sheet models. That is partly a resolution problem, but also the proper physics of ice-sheet dynamics at bases and lateral margins of ice streams and at grounding lines are not yet well known. In particular, treatment of basal sliding, together with the appropriate stresses involved, needs to be incorporated in a more realistic way. Many of these challenges pertain in particular to modelling of the West Antarctic ice sheet, as models based on the shallow-ice approximation perform more satisfactorily for the largely continental-based East Antarctic ice sheet. Clearly, there is much room for improvement of numerical models of the Antarctic ice sheet.

References

Alley, R. B. and Whillans, I. M. 1991. Changes in the West Antarctic ice sheet. *Science* **254**, 959–63.

Anandakrishnan, S. and Alley, R. B. 1997. Stagnation of ice stream C, West Antarctica by water piracy. *Geophys. Res. Lett.* **24** (3), 265–8.

Anandakrishnan, S., Blankenship, D. D., Alley, R. B. and Stoffa, P. L. 1998. Influence of subglacial geology on the position of a West Antarctic ice stream. *Nature* **394**, 62–5.

Barcilon, V. and MacAyeal, D. R. 1993. Steady flow of a viscous ice stream across a no-slip/free-slip transition at the bed. *J. Glaciol.* **39** (131), 167–85.

Barker, P. F., Barrett, P. J., Cooper, A. F. K. and Huybrechts, P. 1999. Antarctic glacial history from numerical models and continental margin sediments. *Palaeogeog., Palaeoclimatol., Palaeoecol.* **150**, 247–67.

Bell R. E. *et al.* 1998. Influence of subglacial geology on the onset of a West Antarctic ice stream from aerogeophysical observations. *Nature* **394**, 58–62.

Bentley, C. R. 1997. Rapid sea-level rise soon from West Antarctic ice sheet collapse? *Science* **275**, 1077–8.

1998a. Ice on the fast track. *Nature* **394**, 21–2.

1998b. Rapid sea-level rise from a West-Antarctic ice-sheet collapse: a short-term perspective. *J. Glaciol.* **44** (146), 157–63.

Blankenship, D. D., Bentley, C. R., Rooney, S. T. and Alley, R. B. 1986. Seismic measurements reveal a saturated porous layer beneath an active Antarctic ice stream. *Nature* **322**, 54–7.

Böhmer, W. J. and Herterich, K. 1990. A simplified 3-D ice sheet model including ice shelves. *Ann. Glaciol.* **14**, 17–19.

Braithwaite, R. J. and Olesen, O. B. 1989. Calculation of glacier ablation from air temperature, west Greenland. In Oerlemans, J., ed., *Glacier Fluctuations and Climatic Change*. Dordrecht, Kluwer Academic Publishers, pp. 219–33.

Bromwich, D. H. 1995. Ice sheets and sea level. *Nature* **373**, 18–19.

Bromwich, D. H., Cullather, R. I. and Van Woert, M. L. 1998. Antarctic precipitation and its contribution to the global sea-level budget. *Ann. Glaciol.* **27**, 220–6.

Budd, W. F. and Smith, I. N. 1982. Large-scale numerical modelling of the Antarctic ice sheet. *Ann. Glaciol.* **3**, 42–9.

Budd, W. F., Jenssen, D. and Radok, U. 1971. *Derived Physical Characteristics of the Antarctic Ice Sheet*. ANARE Interim Report, Series A (IV), Glaciology Publication no. 18, University of Melbourne.

Budd, W. F., Jenssen, D., Mavrakis, E. and Coutts, B. 1994. Modelling the Antarctic ice sheet changes through time. *Ann. Glaciol.* **20**, 291–7.

Budd, W. F., Reid, P. A. and Minty, L. J. 1995. Antarctic moisture flux and net accumulation from global atmospheric analyses. *Ann. Glaciol.* **21**, 149–56.

Budd, W. F., Coutts, B. and Warner, R. C. 1998. Modelling the Antarctic and northern-hemisphere ice-sheet changes with global climate through the glacial cycle. *Ann. Glaciol.* **27**, 153–60.

Church, J. A. *et al.* 2001. Changes in sea level. In Houghton, J. T. et al., eds., *Climate Change 2001: The Scientific Basis*. Cambridge University Press, pp. 639–94.

Clarke, G. K. C., Nitsan, U. and Paterson, W. S. B. 1977. Strain heating and creep instability in glaciers and ice sheets. *Rev. Geophys. & Space Phys.* **15**, 235–47.

Conway, H. W., Hall, B. L., Denton, G. H., Gades, A. M. and Waddington, E. D. 1999. Past and future grounding-line retreat of the West Antarctic ice sheet. *Science* **286**, 280–6.

Doake, C. S. M., Corr, H. F. J., Rott, H., Skvarca, P. and Young, N. W. 1998. Breakup and conditions for stability of the northern Larsen ice shelf, Antarctica. *Nature* **391**, 778–80.

Echelmeyer, K. A., Harrison, W. D., Larsen, C. and Mitchell, J. E. 1994. The role of the margins in the dynamics of an active ice stream. *J. Glaciol.* **40** (136), 527–38.

Fastook, J. L. and Prentice, M. L. 1994. A finite-element model of Antarctica: sensitivity test for meteorological mass balance relationship. *J. Glaciol.* **40** (134), 167–75.

Fortuin, J. P. F. and Oerlemans, J. 1990. Parameterisation of the annual surface temperature and mass balance of Antarctica. *Ann. Glaciol.* **14**, 78–84.

Giovinetto, M. B. and Zwally, H. J. 1995a. An assessment of the mass budgets of Antarctica and Greenland using accumulation derived from remotely sensed data in areas of dry snow. *Zeits. Gletscherkunde & Glazialgeol.* **31**, 25–37.

1995b. Annual changes in sea ice extent and of accumulation on ice sheets: implications for sea level variability. *Zeits. Gletscherkunde & Glazialgeol.* **31**, 39–49.

2000. Spatial distribution of net surface accumulation on the Antarctic ice sheet. *Ann. Glaciol.* **31**, 171–8.

Glen, J. W. 1955. The creep of polycrystalline ice. *Proc. Roy. Soc. London Series B* **228**, 519–38.

Greve, R. and MacAyeal, D. R. 1996. Dynamic/thermodynamic simulations of Laurentide ice sheet instability. *Ann. Glaciol.* **23**, 328–35.

Greve, R., Mügge, B., Baral, D. R., Albrecht, O. and Savvin, A. 1999. Nested high-resolution modelling of the Greenland Summit Region. In Hutter, K., Wang, Y. and Beer, H., eds., *Advances in Cold-Region Thermal Engineering and Sciences*. Springer Verlag, pp. 285–306.

Herterich, K. 1987. On the flow within the transition zone between ice sheet and ice shelf. In Van der Veen, C. J. and Oerlemans, J., eds., *Dynamics of the West Antarctic Ice Sheet*. Dordrecht, D. Reidel, pp. 185–202.

1988. A three-dimensional ice-sheet model of the Antarctic ice sheet. *Ann. Glaciol.* **11**, 32–5.

Hindmarsh, R. C. A. 1993. Qualitative dynamics of marine ice sheets. In Peltier, W. R., ed., *Ice in the Climate System*. NATO ASI Series I12, pp. 68–99.

Hughes, T. J. 1975. The West Antarctic ice sheet: instability, disintegration, and initiation of ices ages. *Rev. Geophys. & Space Phys.* **13**, 502–26.

Hulbe, C. L. 1998. Heat balance of West Antarctic ice streams, investigated with a numerical model of coupled ice sheet, ice stream and ice shelf flow. Ph.D. thesis, University of Chicago.
Hulbe, C. L. and MacAyeal, D. R. 1999. A new numerical model of coupled inland ice sheet, ice stream, and ice shelf flow and its application to the West Antarctic ice sheet. *J. Geophys. Res.* **104** (B11), 25 349–66.
Hulbe, C. L. and Payne, A. J. 2001. The contribution of numerical modelling to our understanding of the West Antarctic ice sheet. In Alley, R. B. and Bindschadler, R. A., eds. *The West Antarctic Ice Sheet: Behaviour and Environment*. Antarctic Research Series, 77, Washington D. C., American Geophysical Union, pp. 201–19.
Hutter, K. 1983. *Theoretical Glaciology*. Dordrecht, D. Reidel.
Huybrechts, P. 1990a. A 3-D model for the Antarctic ice sheet: a sensitivity study on the glacial–interglacial contrast. *Climate Dyn.* **5**, 79–92.
1990b. The Antarctic ice sheet during the last glacial-interglacial cycle: a three dimensional experiment. *Ann. Glaciol.* **11**, 52–9.
1992. *The Antarctic Ice Sheet and Environmental Change: A Three-Dimensional Modeling Study*. Berichte zur Polarforschung 99, Bremerhaven, Alfred-Wegener-Institut für Polar- und Meeresforschung.
1994. Formation and disintegration of the Antarctic ice sheet. *Ann. Glaciol.* **20**, 336–40.
2002. Sea-level changes at the LGM from ice-dynamic reconstructions of the Greenland and Antarctic ice sheets during the glacial cycles. *Quat. Sci. Rev.* **21** (1–3), 203–31.
Huybrechts, P. and De Wolde, J. 1999. The dynamic response of the Greenland and Antarctic ice sheets to multiple-century climatic warming. *J. Climate* **12** (8), 2169–88.
Huybrechts, P. and Le Meur, E. 1999. Predicted present-day evolution patterns of ice thickness and bedrock elevation over Greenland and Antarctica. *Polar Res.*, **18** (2), 299–308.
Huybrechts, P. and Oerlemans, J. 1988. Evolution of the East Antarctic ice sheet: a numerical study of thermo-mechanical response patterns with changing climate. *Ann. Glaciol.* **11**, 52–9.
1990. Response of the Antarctic ice sheet to future greenhouse warming. *Climate Dyn.* **5**, 93–102.
Huybrechts, P. *et al.* 1996. The EISMINT benchmarks for testing ice-sheet models. *Ann. Glaciol.* **23**, 1–12.
Huybrechts, P. *et al.* 1998. *Report of the Third EISMINT Workshop on Model Intercomparison*. Strasbourg, European Science Foundation.
Huybrechts, P., Steinhage, D., Wilhelms, F. and Bamber, J. L. 2000. Balance velocities and measured properties of the Antarctic ice sheet from a new compilation of gridded datasets for modeling. *Ann. Glaciol.* **30**, 52–60.
Imbrie, J. Z. *et al.* 1984. The orbital theory of Pleistocene climate: support from a revised chronology of the marine δ18O record. In Berger, A. *et al.*, eds., *Milankovitch and Climate*. Dordrecht, D. Reidel, pp. 269–305.
Ingolfsson, O. *et al.* 1998. Antarctic glacial history since the last glacial maximum: an overview of the record on land. *Antarctic Sci.* **10** (3), 326–44.
Jacobs, S. J., Hellmer, H. H. and Jenkins, A. 1996. Antarctic ice sheet melting in the Southeast Pacific. *Geophys. Res. Lett.* **23** (9), 957–60.
Jacobson, H. P. and Raymond, C. F. 1998. Thermal effects on the location of ice stream margins. *J. Geophys. Res.* **103** (B6), 12 111–22.

James, T. S. and Ivins, E. R. 1998. Predictions of Antarctic crustal motions driven by present-day ice sheet evolution and by isostatic memory of the last glacial maximum. *J. Geophys. Res.* **103** (B3), 4993–5017.

Jenkins, A., Vaughan, D. G., Jacobs, S. J., Hellmer, H. H. and Keys, J. R. 1997. Glaciological and oceanographic evidence of high melt rates beneath Pine Island Glacier, West Antarctica. *J. Glaciol.* **43** (143), 114–21.

Jenssen, D. 1977. A three-dimensional polar ice sheet model. *J. Glaciol.* **18** (80), 373–89.

Kotlyakov, V. M., Losev, K. S. and Loseva, I. A. 1978. The ice budget of Antarctica. *Polar Geog. & Geol.* **2** (4), 251–62.

Krinner, G., Genthon, C., Li, Z. X. and Le Van, P. 1997. Studies of the Antarctic climate with a stretched-grid general circulation model. *J. Geophys. Res.* **102** (D12), 13 731–45.

Le Meur, E. and Huybrechts, P. 1996. A comparison of different ways of dealing with isostasy: examples from modeling the Antarctic ice sheet during the last glacial cycle. *Ann. Glaciol.* **23**, 309–17.

Lestringant, R. 1994. A 2D finite element study of the flow in the transition zone between an ice sheet and an ice shelf. *Ann. Glaciol.* **20**, 67–72.

Lingle, C. S. 1984. A numerical model of interactions between a polar ice stream and the ocean: application to ice stream E, West Antarctica. *J. Geophys. Res.* **89**, 3524–49.

Lorius, C. *et al.* 1985. A 150 000-year climatic record from Antarctic ice. *Nature* **316**, 591–6.

MacAyeal, D. R. 1992. Irregular oscillations of the West Antarctic ice sheet. *Nature* **359**, 29–32.

MacAyeal, D. R. and Thomas, R. H. 1986. The effects of basal melting on the present flow of the Ross ice shelf, Antarctica. *J. Glaciol.* **32** (110), 72–86.

MacAyeal, D. R., Rommelaere, V., Huybrechts, P., Hulbe, C. L., Determann, J. and Ritz, C. 1996. An ice-shelf model test based on the Ross ice shelf. *Ann. Glaciol.* **23**, 46–51.

Mahaffy, M. A. W. 1976. A three-dimensional numerical model of ice sheets: tests on the Barnes ice cap, Northwest Territories. *J. Geophys. Res.* **81** (6), 1059–66.

Manabe, S., Stouffer, R. J., Spelman, M. J. and Bryan, K. 1991. Transient response of a coupled ocean-atmosphere model to gradual changes of atmospheric CO_2. Part I: Annual mean response. *J. Climate* **4** (8), 785–818.

Mayer, C. and Huybrechts, P. 1999. Ice-dynamic conditions across the grounding zone, Ekströmisen, East Antarctica. *J. Glaciol.* **45** (150), 384–93.

Mercer, J. H. 1978. West Antarctic ice sheet and CO_2 greenhouse effect: a threat of disaster. *Nature* **271**, 321–5.

Mosley-Thompson, E., Paskievitch, J. F., Gow, A. J. and Thompson, L. G. 1999. Late 20th century increase in South Pole accumulation. *J. Geophys. Res.* **104** (D4), 3877–86.

Nicholls, K. W. 1997. Predicted reduction in basal melt rates of an Antarctic ice shelf in a warmer climate. *Nature* **388**, 460–2.

Nye, J. F. 1957. The distribution of stress and velocity in glaciers and ice sheets. *Proc. Roy. Soc. London Series A* **239**, 113–33.

Oerlemans, J. 1982a. A model of the Antarctic ice sheet. *Nature* **297** (5967), 550–3.

1982b. Response of the Antarctic ice sheet to a climatic warming: a model study. *J. Climatol.* **2**, 1–11.

1983. A numerical study on cyclic behaviour of polar ice sheets. *Tellus* **35A**, 81–7.

Oerlemans, J. and Van der Veen, C. J. 1984. *Ice Sheets and Climate*. Dordrecht, D. Reidel.

Oerter, H. *et al.* 1992. Evidence for basal marine ice in the Filchner-Ronne ice shelf. *Nature* **358**, 399–401.

O'Farrell, S. P., McGregor, J. L., Rotstayn, L. D., Budd, W. F., Zweck, C. and Warner, R. C. 1997. Impact of transient increases in atmospheric CO_2 on the accumulation and mass balance of the Antarctic ice sheet. *Ann. Glaciol.* **25**, 137–44.

Ohmura, A., Wild, M. and Bengtsson, L. 1996. Present and future mass balance of the ice sheets simulated with GCM. *Ann. Glaciol.* **23**, 187–93.

Oppenheimer, M. 1998. Global warming and the stability of the West Antarctic ice sheet. *Nature* **393**, 325–32.

Pattyn, F. 1996. Numerical modelling of a fast flowing outlet glacier: experiments with different basal conditions. *Ann. Glaciol.* **23**, 237–46.

Payne, A. J. 1995. Limit cycles in the basal thermal regime of ice sheets. *J. Geophys. Res.* **100** (B3), 4249–63.

1999. A thermomechanical model of ice flow in West Antarctica. *Climate Dyn.* **15**, 115–25.

Payne, A. J. *et al.* 2000. Results from the EISMINT Phase 2 simplified geometry experiments: the effects of thermomechanical coupling. *J. Glaciol.* **46** (153), 227–38.

Peel, D. A. and Mulvaney, R. 1988. Air temperature and snow accumulation in the Antarctic Peninsula during the past 50 years. *Ann. Glaciol.* **11**, 206–7.

Petit, J. R. *et al.* 1999. Climate and atmospheric history of the past 420 000 years from the Vostok ice core, Antarctica. *Nature* **399**, 429–36.

Reeh, N. 1991. Parameterisation of melt rate and surface temperature on the Greenland ice sheet. *Polarforschung* **59**, 113–28.

Retzlaff, R. and Bentley, C. R. 1993. Timing of stagnation of ice stream C, West Antarctica, from short-pulse radar studies of buried surface crevasses. *J. Glaciol.* **39** (133), 553–61.

Ritz, C., Rommelaere, V. and Dumas, C. 2001. Modeling the evolution of the Antarctic ice sheet over the last 420000 years: implications for altitude changes in the Vostok region. *J. Geophys. Res.* **106** (D23), 31 943–64.

Robin, G. de Q. 1977. Ice cores and climatic change. *Phil. Trans. Roy. Soc. Lond. A.* **280**, 143–68.

Rommelaere, V. and MacAyeal, D. R. 1997. Large-scale rheology of the Ross ice shelf, Antarctica, computed by a control method. *Ann. Glaciol.* **24**, 43–8.

Savvin, A., Greve, R., Calov, R., Mügge, B. and Hutter, K. 2000. Simulation of the Antarctic ice sheet with a three-dimensional polythermal ice-sheet model, in support of the EPICA project. II: Nested high-resolution treatment of Dronning Maud Land, Antarctica. *Ann. Glaciol.* **30**, 69–75.

Schubert, G. and Yuen, D. A. 1982. Initiation of ice ages by creep instability and surging of the East Antarctic ice sheet. *Nature* **296**, 127–30.

Skvarca, P., Rack, W., Rott, H. and Ibarzabal y Donangelo, T. 1998. Evidence of recent climatic warming on the eastern Antarctic Peninsula. *Ann. Glaciol.* **27**, 628–32.

Smith, I. N., Budd, W. F. and Reid, P. 1998. Model estimates of Antarctic accumulation rates and their relationship to temperature changes. *Ann. Glaciol.* **27**, 246–50.

Steig, E. J. 1997. How well can we parameterize past accumulation rates in polar ice sheets? *Ann. Glaciol.* **25**, 418–22.

Stephenson, S. N. and Bindschadler, R. A. 1988. Observed velocity fluctuations on a major Antarctic ice stream. *Nature* **334**, 695–7.

Thomas, R. H. 1979. The dynamics of marine ice sheets. *J. Glaciol.* **24** (90), 167–77.

Thomas, R. H. and Bentley, C. R. 1978. A model for Holocene retreat of the West Antarctic ice sheet. *Quat. Res.* **10**, 150–70.

Thomas, R. H., Sanderson, T. J. O. and Rose, K. E. 1979. Effect of climatic warming on the West Antarctic ice sheet. *Nature* **277**, 355–8.
Thompson, S. L. and Pollard, D. 1997. Greenland and Antarctic mass balances for present and doubled atmospheric CO_2 from the GENESIS version-2 global climate model. *J. Climate* **10**, 871–900.
Turner, J., Connolley, W. M., Leonard, S., Marshall, G. J. and Vaughan, D. G. 1999. Spatial and temporal variability of net snow accumulation over the Antarctic from ECMWF re-analysis project data. *Int. J. Climatol.* **19**, 697–724.
Tushingham, A. M. and Peltier, W. R. 1991. Ice-3G: a new global model of Late Pleistocene deglaciation based upon geophysical predictions of post-glacial relative sea level change. *J. Geophys. Res.* **96** (B3), 4497–523.
Van der Veen, C. J. 1985. Response of a marine ice sheet to changes at the grounding line. *Quat. Res.* **24**, 257–67.
1986. Ice sheets, atmospheric CO_2 and sea level. Ph.D. Thesis, University of Utrecht.
Van Lipzig, N. P. M. 1999. The surface mass balance of the Antarctic ice sheet: a study with a regional atmospheric model. Ph.D. thesis, University of Utrecht.
Vaughan, D. G. and Doake, C. S. M. 1996. Recent atmospheric warming and retreat of ice shelves on the Antarctic Peninsula. *Nature* **379**, 328–31.
Vaughan, D. G. and Spouge, J. 2002. Risk estimation of collapse of the West Antarctic ice sheet. *Climatic Change* **52**, 65–91.
Vaughan, D. G., Bamber, J. L., Giovinetto, M. B., Russell, J. and Cooper, A. P. R. 1999. Reassessment of net surface mass balance in Antarctica. *J. Climate* **12**, 933–46.
Warner, R. C. and Budd, W. F. 1998. Modelling the long-term response of the Antarctic ice sheet to global warming. *Ann. Glaciol.* **27**, 161–8.
Weertman, J. 1957. Deformation of floating ice shelves. *J. Glaciol.* **3**, 38–42.
1974. Stability of the junction of an ice sheet and an ice shelf. *J. Glaciol.* **13** (67), 3–11.
Wild, M. and Ohmura, A. 2000. Changes in mass balance of the polar ice sheets and sea level under greenhouse warming as projected in high resolution GCM simulations. *Ann. Glaciol.* **30**, 197–203.
Williams, M. J. M., Jenkins, A. and Determann, J. 1998. Physical controls on ocean circulation beneath ice shelves revealed by numerical models. In Jacobs, S. J. and Weiss, R. F., eds., *Ocean, Ice, and Atmosphere: Interactions at the Antarctic Continental Margin*. Antarctic Research Series 75. Washington D.C., pp. American Geophysical Union, pp. 285–99.
Williams, M. J. M., Warner, R. C. and Budd, W. F. 1998. The effects of ocean warming on melting and ocean circulation under the Amery ice shelf, East Antarctica. *Ann. Glaciol.* **27**, 75–80.
Wingham, D. J., Ridout, A. J., Scharroo, R., Arthern, R. J. and Shum, C. K. 1998. Antarctic elevation change from 1992 to 1996. *Science* **282**, 456–8.
Yiou, F., Raisbeck, G. M., Bourles, D., Lorius, C. and Barkov, N. I. 1985. ^{10}Be in ice at Vostok Antarctica during the last climatic cycle. *Nature* **316**, 616–17.
Yuen, D. A., Saari, M. R. and Schubert, G. 1986. Explosive growth of shear-heating instabilities in the down-slope creep of ice sheets. *J. Glaciol.* **32** (112), 314–20.