Estimation of Effective Vertical Diffusivity in Turbulent Exchange Flows by Numerical Experiments

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The importance of exchange flows through channels or straits, which are interconnecting water bodies of different densities, lies in the fact that water mass characteristics in sub-basins are determined by these flows, extend of which depends on the exchange rate between the interconnected water bodies. The knowledge of vertical diffusivity, κ, along a channel/strait is therefore crucial as this gives us an idea about the amount of heat, mass and dissolved solids transported across it and provide parameterisations for circulation models. From comparison with numerical results of an exchange through a channel at various values for Gr, Pr and aspect ratio, we infer the effective value for κ as it appears in the Grashof number.

§1. Introduction

Goitsche Lake Complex in Central Germany comprises two sub-basins, namely Mühlbeck and Niemegk, with distinct water compositions, which are linked to one another by a channel. The channel owes its nearly constant cross section and flat design of the bottom to the mining activity, which left a void after closure of the mine in the 1990s.

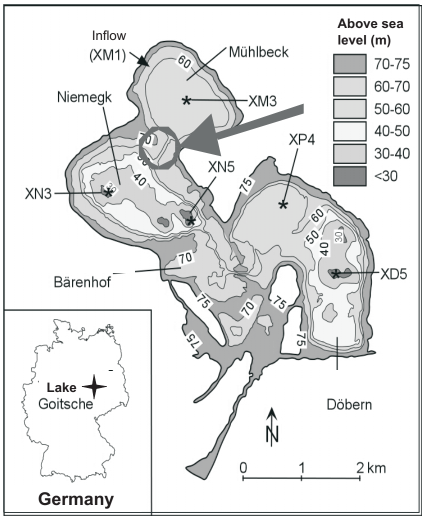

Fig. 1. Goitsche lake.

The level of vertical mixing in stratified fluid systems is parameterised by vertical diffusivity. In this study we used a simplistic approach, that is scaling analysis, to define a relation between non-dimensional parameters specifying the thickness of the intrusion layer between two oppositely flowing layers in density driven through-flows. The parameters that are used are the Grashof number, $Gr = g\prime H^3/\nu^2$, the Prandtl number, $Pr = \nu/\kappa$, and the aspect ratio, $A = H/L$, where $g\prime = g\Delta\rho/\rho_0$ is the reduced gravity, $\Delta\rho$ is the density difference between the two ends of the channel, ρ_0 is the reference density, ν is the kinematic viscosity, κ is the vertical diffusivity, H is the height of the channel, and L is the length of the channel.

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§2. Methods

We argue that the time scale for vertical mixing between two oppositely flowing layers is specified by the time for the flow to travel along the channel. Therefore we can write the nondimensional thickness of the intrusion layer dependent on its thermal conductivity in this time scale, t, as shown by Boehrer[1] to be valid for the intrusion layers in the long cavity:

$$h \sim \frac{(t\kappa)^{1/2}}{H}, \tag{2·1}$$

where

$$t \sim \frac{L}{(g\prime H)^{1/2}}. \tag{2·2}$$

Since $Gr = g\prime H^3/\nu^2$, the interface thickness then scales with

$$h \sim (GrA^2)^{-1/4} Pr^{-1/2}. \tag{2·3}$$

By carrying out a number of numerical experiments with varying Grashof number, Prandtl number and aspect ratio, we, herein, give an estimation of the effective value for κ in the Goitsche channel. Although a similar scaling argument was put forward by Hogg et al.[2] their work did not take into account the effect of different Pradtl numbers on the flow.

We solve Navier-Stokes equations describing mass, Eq. (2·4), momentum, Eq. (2·5), and heat, Eq. (2·6), transfer in three dimensions:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho U) = 0, \tag{2·4}$$

$$\frac{\partial \rho U}{\partial t} + \nabla \cdot (\rho U \otimes U) = \nabla \cdot (-p\delta + \mu(\nabla U + (\nabla U)^T)) + S_M, \tag{2·5}$$

$$\frac{\partial \rho h_{\mathrm{tot}}}{\partial t} - \frac{\partial p}{\partial t} + \nabla \cdot (\rho U h_{\mathrm{tot}}) = \nabla \cdot (\lambda \nabla T) + \Big(\nabla \cdot \Big(\mu\Big(\nabla U + (\nabla U)^T - \frac{2}{3}\nabla \cdot U\delta\Big)U\Big) + S_E\Big), \tag{2·6}$$

where, ρ is density, $\mu = \rho\nu$ is molecular (dynamic) viscosity, $\lambda = \rho C_p \kappa$ is thermal conductivity, C_p is heat capacity, U is vector of velocity, S_E is energy source, p is static pressure, δ is identity matrix, T is static temperature, S_M is momentum source and h_{tot} is specific total enthalpy.

The numerical calculations were carried out using the general-purpose code, CFX-5.[3] The code employs a finite volume approach method. The solution variables and fluid properties are stored at the element nodes, which are surrounded by a set of surfaces containing finite volume. Boussinesq approximation was applied for the buoyancy force. The water properties were constant apart from the density variation with temperature.

Conventional upwind difference scheme with numerical advection correction is used for the advection terms in the momentum and energy equations. The scheme is second order accurate when the numerical advection corrector is fully introduced. A fourth order 'pressure redistribution' term is introduced in the discretised equations to control the pressure-velocity coupling in the mass and momentum equations. An improved version of Rhie-Chow interpolation method is used to overcome the checker-board oscillations in pressure and velocity. Meshing was based on uniform tetrahedral/triangular element discretisation. There were 30 grid cells along the channel height, H.

§3. Results

We conducted 10 numerical experiments as listed in Table I. The tested aspect ratios were 0.03, 0.05 and 0.1. Two different Prandtl numbers were tried, 7 and 143, the latter being the typical of sea water, while the Grashof number varied from $8.1{\cdot}10^3$ up to $16.5{\cdot}10^5$. Figure 2 illustrates the longitudinal slices through the model domain for the two of these simulations.

Table I. Numerical experiments.

Exp. no	A	Pr	Gr
I	0.1	7	$810{\cdot}10^3$
II	0.1	7	$160{\cdot}10^3$
III	0.1	7	$91{\cdot}10^3$
IV	0.1	7	$41{\cdot}10^3$
V	0.1	143	$8.1{\cdot}10^3$
VI	0.1	143	$41{\cdot}10^3$
VII	0.1	143	$410{\cdot}10^3$
VIII	0.05	7	$1600{\cdot}10^3$
IX	0.03	7	$1600{\cdot}10^3$
X	0.03	7	$82{\cdot}10^3$

We obtained a range of thicknesses of intrusion layer from numerical experiments. In Fig. 3 the variation of the interface thicknesses is plotted against $(GrA^2)^{-1/4}\ Pr^{-1/2}$ in order to find out the slope of scaling argument presented in Eq. (2·3). The interface thickness is specified as the distance between points where $\rho = \rho_0 - 0.4\Delta\rho$ and $\rho = \rho_0 + 0.4\Delta\rho$. The linear regression of the data gives a constant of approximately 1.11. Thus,

$$h = 1.11(GrA^2)^{-1/4}Pr^{-1/2}. \qquad (3{\cdot}1)$$

Finally, we show in Fig. 4 three density profiles calculated from the temperature and conductivity measurements along the channel in July 2000, along with the profiles taken from the sub-basins. The nondimensional interface thickness in these profiles is found as around 0.0711 of the channel depth. Now, by using the relation in Eq. (3·1) in the reverse order, having the values of 500 m for the channel length and 5 m for the height and assuming $Pr = 1$, we find out the estimated value of κ

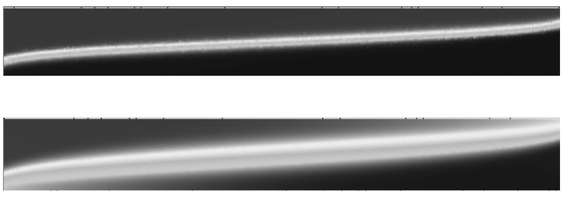

Fig. 2. Simulations with $Pr = 143$, $A = 0.1$, $Gr = 4.1{\cdot}10^5$ (top) and $Pr = 7$, $A = 0.1$, $Gr = 4.1{\cdot}10^4$ (bottom)(shades refer to different temperatures).

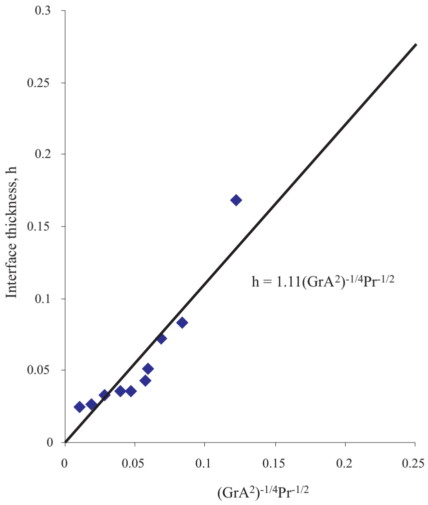

Fig. 3. h versus $(GrA^2)^{-1/4}Pr^{-1/2}$.

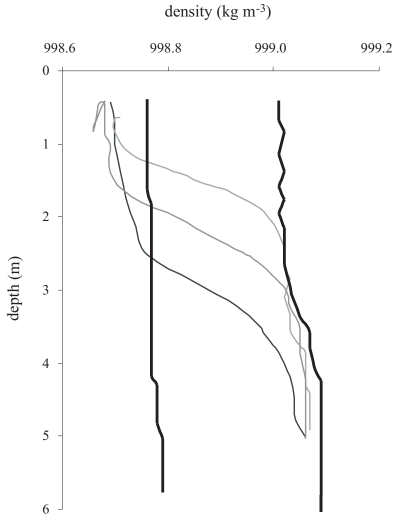

Fig. 4. Density profiles at three locations along the channel.

for the Goitsche channel as $9{\cdot}10^{-4}\mathrm{m}^2\mathrm{s}^{-1}$.

This value of κ seems high and needs comparison. Hogg et al.[2] give in their argument, which does not involve Pr number, an estimation of GrA^2 values for several channels and sea straits. By assuming that $\nu{=}\kappa$, i.e. transport is controlled by turbulent processes, which indicates that momentum and properties contributing to density are transported at the same rate, GrA^2 value for Goitsche channel is calculated from Eq. (3·1) as $5{\cdot}10^4$. This is very similar to the GrA^2 value of Burlington Ship Canal, a channel between Hamilton Harbour and the main body of Lake Ontario, which amounts to $3{\cdot}10^4$. Burlington Ship Channel has the same aspect ratio as our study site with a 9 m depth and 850 m length. On the basis of density measurements by Hamblin et al.[4] we find the κ value of the channel as $10^{-2}\mathrm{m}^2\mathrm{s}^{-1}$, which is a factor of 10 times even larger than our result, although the density difference during Hamblin's measurements was 8 times higher than Goitsche channel.

Acknowledgements

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