On the validity of the Millionshchikov quasi-normality hypothesis for open-ocean deep convection

Martin Losch

Alfred-Wegener-Institut für Polar- und Meeresforschung, Bremerhaven, Germany

Data from numerical simulations of free convection in the ocean with rotation are used to explore the validity of the Millionshchikov quasi-normality hypothesis. The parameterizations of fourth-order moments according to a universal model proposed by Gryanik et al. are found to be far more accurate than their corresponding Gaussian parameterizations, which are based on the Millionshchikov hypothesis. The universal model is marginally improved by fitting the model parameters to the data. The present results extend previous results that discuss shallow convection in the planetary boundary layer to rotationally controlled deep convection.

1. Introduction

Turbulent vertical mixing in ocean models is generally parameterized by bulk mixed-layer models [e.g., *Krauss and Turner*, 1967] or first order [e.g., *Pacanowski and Philander*, 1981; *Large et al.*, 1994] or second-order [e.g., *Mellor and Yamada*, 1982] turbulence closure models. For deep convection, the parameterizations are usually achieved by even cruder methods, for example the so-called "convective adjustment" or high vertical diffusivities in the case of unstable stratification [*Haidvogel and Beckmann*, 1999]. Recently, *Canuto et al.* [2001b] formulated a turbulence closure model for the ocean that advances the traditional one-point closure models, in particular the second-order closure model of *Mellor and Yamada* [1982], by an improved treatment of pressure correlations. Higher-order closures make further advances in mixed layer modeling possible. For example, *Canuto et al.* [2001b] presented an expression for third-order moments (TOM), that implicitly makes use of the Millionshchikov hypothesis to represent fourth-order moments. According to the Millionshchikov hypothesis [*Millionshchikov*, 1941; *Monin et al.*, 1971], fourth-order moments in higher-order closure (HOC) models can be approximated as quasi-normal (Gaussian), that is, by a combination of second-order moments,

$$\langle ABCD\rangle = \langle AB\rangle\langle CD\rangle + \langle AC\rangle\langle BD\rangle + \langle AD\rangle\langle BC\rangle, \quad (1)$$

even if the third-order moments are non-zero. $\langle\cdot\rangle$ denotes the time average.

Recent studies found evidence that the Millionshchikov hypothesis fails for vertical velocity and temperature in the atmospheric turbulent convective boundary layer (CBL) [*Andre et al.*, 1976; *Moeng and Randall*, 1984; *Canuto et al.*, 2001a; *Gryanik and Hartmann*, 2002; *Alberghi et al.*, 2002; *Gryanik et al.*, 2005]. One of the problems is that a model based on the quasi-normality hypothesis does not include the effects of coherent structures [*Salmon*, 1998] typical of convective regimes. *Gryanik and Hartmann* [2002] and *Gryanik et al.* [2005] explained the failure of the Millionshchikov hypothesis by the skewed nature of the CBL turbulence with respect to upward-downward and hot-cold fluctuations. They derived non-Gaussian parameterizations for the fourth-order moments of temperature and vertical velocity that explicitly account for the skewness of the velocity and temperature fluctuations due to semi-organized coherent structures (plumes) in turbulent convection.

Oceanic deep convection is characterized by plumes on horizontal scales less than 1 km. The convective plumes are affected by rotation and lateral entrainment [*Marshall and Schott*, 1999]. The presence of plumes already indicates a non-normal distribution for vertical velocity and density. In this note, we investigate the validity of the quasi-normal approximation of fourth-order moments for an oceanic convective regime typical for the Greenland or Labrador Seas, but without lateral mixing. Further, we demonstrate how the refinement proposed by *Gryanik and Hartmann* [2002] and *Gryanik et al.* [2005] improves the parameterization of fourth-order moments $\langle w'^4\rangle$, $\langle\theta'^4\rangle$, $\langle w'^3\theta'\rangle$, $\langle w'\theta'^3\rangle$, and $\langle w'^2\theta'^2\rangle$ of the temperature θ and vertical velocity w in this convective regime. Because observations of mixing and convection, particularly of higher-order moments, in the ocean are sparse or not available, we use a numerical non-hydrostatic ocean model, the MITgcm, to simulate convection in the ocean in an LES fashion. The MITgcm [*Marshall et al.*, 1997] has been used successfully for simulations of deep convection [*Jones and Marshall*, 1993, 1997] and internal mixing [*Legg and Adcroft*, 2003].

2. Numerical Experiments

2.1. Experiment configuration

For computational efficiency we consider only the two-dimensional case. Full three-dimensional convection will be treated in a subsequent study. The computational domain represents a horizontal section through the ocean in the x-z-plane. The domain is doubly periodic, 3 km wide, and 1 km deep; the grid spacing is 10 m and uniform. Gravity acceleration is $g = 9.81\,\mathrm{m\,s^{-2}}$.

In order to assess the reliability of the results, different approximations of subgrid processes were used: harmonic dissipation with and without variable viscosity or biharmonic dissipation of momentum; harmonic or biharmonic diffusion for tracers. In all cases, many different choices of viscosity and diffusivity parameters were used. While the different choices certainly lead to different realizations of the flow field, the statistical properties of the solution are so robust that the presented results do not depend on the particular form of subgrid dissipation. Therefore we show only one example, in which horizontal viscosity is parameterized following *Leith* [1968] with a maximum viscosity parameter of $A_h^{\max} = 10^{-1}\,\mathrm{m^2\,s^{-1}}$. In the vertical the viscosity parameter is $A_v = 10^{-3}\,\mathrm{m^2\,s^{-1}}$. For scalar tracers we choose a third-order direct space-time advection scheme [*Hundsdorfer et al.*, 1995]. This advection scheme is stable without explicit diffusion, so that the horizontal and vertical diffusivities can be set to zero.

More generally, the fixed grid size and the lack of a sophisticated subgrid model imposes a further limitation: the numerical simulation data may not be accurate near the surface and the bottom of the boundary layer because eddies in those regions are too small to be resolved.

In all experiments, constant surface heat flux Q out of the ocean is applied over the time of the integration. Surface heat losses of 200, 400, and $800\,\mathrm{W m^{-2}}$ are chosen to represent typical buoyancy loss values in the Greenland and Labrador Seas [*Marshall and Schott*, 1999]. The Coriolis parameter ranges from $f = 1.4\times10^{-4}\,\mathrm{s^{-1}}$ in most experiments to $f = 0.7\times10^{-4}\,\mathrm{s^{-1}}$. The ocean is governed by a linear equation of state

$$\rho = \rho_0\left[1 - \alpha(\theta - \theta_0)\right],$$

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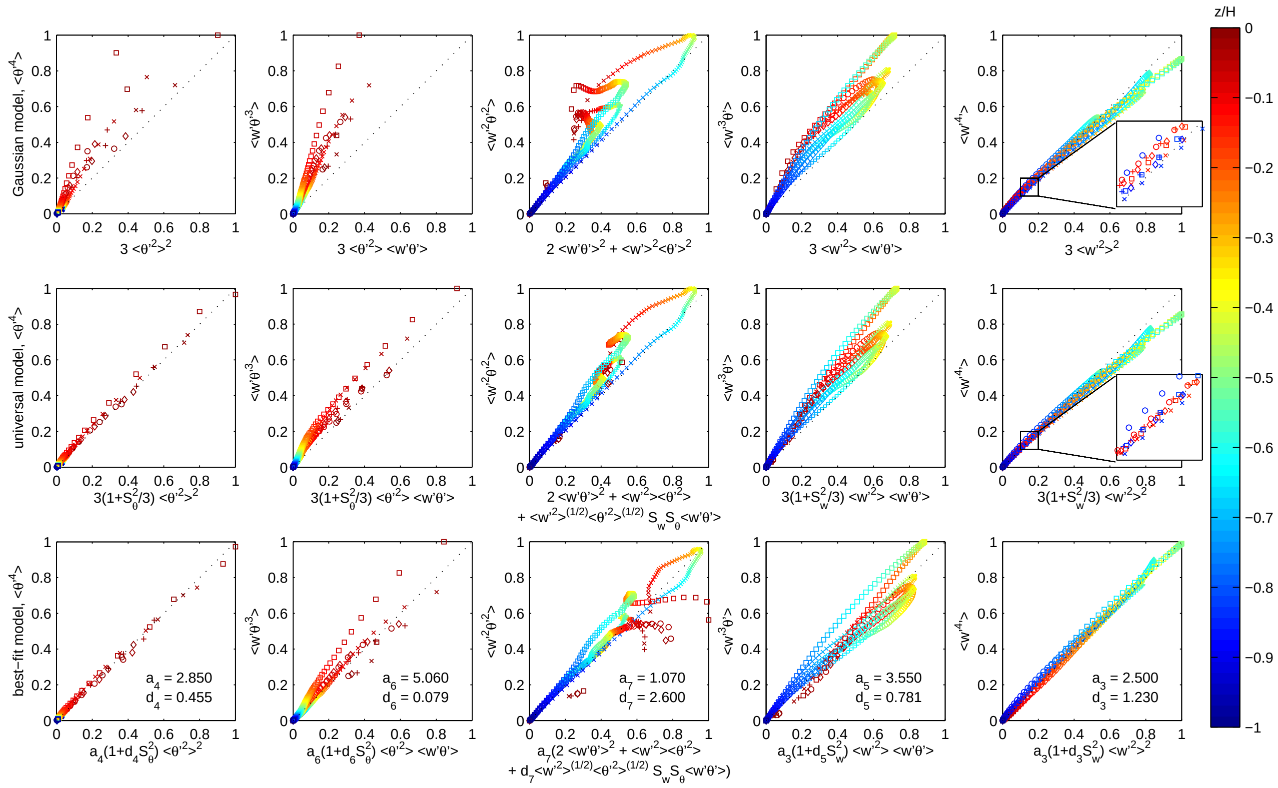

Figure 1. Fourth-order moments plotted against their respective parameterization according to the Gaussian model (Millionshchikov hypothesis, top row), the universal model of *Gryanik and Hartmann* [2002] (middle row), and with the best-fit parameters a_i and d_i (bottom row) (Inset: zoom to a range of 0.1 to 0.2). All values are scaled by the corresponding powers of $w_* = (Q/f)^{1/2}$ and $\theta_* = (Qf)^{1/2}$ and then normalized by the maximum value for plotting. Color indicates the scaled z-coordinate. The different symbols denote data from different experiments: EXP1 (○): $Q = 200\,\mathrm{Wm}^{-2}$, $f = 1.4\times10^{-4}\,\mathrm{s}^{-1}$; EXP2 (+): $Q = 400\,\mathrm{Wm}^{-2}$, $f = 1.4\times10^{-4}\,\mathrm{s}^{-1}$; EXP3 (×): $Q = 800\,\mathrm{Wm}^{-2}$, $f = 1.4\times10^{-4}\,\mathrm{s}^{-1}$; EXP4 (◇): $Q = 200\,\mathrm{Wm}^{-2}$, $f = 1.3\times10^{-4}\,\mathrm{s}^{-1}$; EXP5 (□): $Q = 200\,\mathrm{Wm}^{-2}$, $f = 0.7\times10^{-4}\,\mathrm{s}^{-1}$.

where $\alpha = 2\times10^{-4}\,\mathrm{K}^{-1}$ is the coefficient of thermal expansion of water, and $\rho_0 = 1035\,\mathrm{kg\,m}^3$ and $\theta_0 = 1\,^\circ\mathrm{C}$ are constant reference density and temperature, respectively. The simulations presented here do not contain any haline effects. The initial temperature field of $0.1\,^\circ\mathrm{C}$ is perturbed with random noise of small amplitude ($0.01\,^\circ\mathrm{C}$) to start the convection. Then the model is run for 96 hours. The system reaches a statistically stationary regime after approximately 24 hours. Vertical velocities reach the order of $10\,\mathrm{cm/s}$ so that the time for a water parcel to travel from the surface to the bottom is on the order of a few hours. The moments of temperature and vertical velocity scale with the surface heat flux as $\theta_* = (Qf)^{1/2}$ and $w_* = (Q/f)^{1/2}$, respectively, as predicted for rotationally controlled open-ocean free convection [*Golitsyn*, 1980; *Fernando et al.*, 1991; *Jones and Marshall*, 1993]. The following analyses are based on 48 hour averages starting at 48 hours.

2.2. Testing the Millionshchikov hypothesis

The top row of Fig. 1 shows the fourth-order moments $\langle w'^4\rangle$, $\langle\theta'^4\rangle$, $\langle w'^3\theta'\rangle$, $\langle w'\theta'^3\rangle$, and $\langle w'^2\theta'^2\rangle$ of model simulations with surface heat losses of 200, 400, and $800\,\mathrm{Wm}^2$, and Coriolis parameters of 1.4, 1.3, and $0.7\times10^{-4}\,\mathrm{s}^{-1}$ plotted against their respective Gaussian parameterizations (Eq. 1). Note that the Gaussian parameterization is good for $\langle w'^4\rangle$, but generally underestimates the remaining four moments.

2.3. Comparison to fourth-order moments expressions by *Gryanik and Hartmann* [2002] *Gryanik et al.* [2005]

Gryanik and Hartmann [2002] and *Gryanik et al.* [2005] assume that the skewness is a measure of deviation from Gaussian statistics

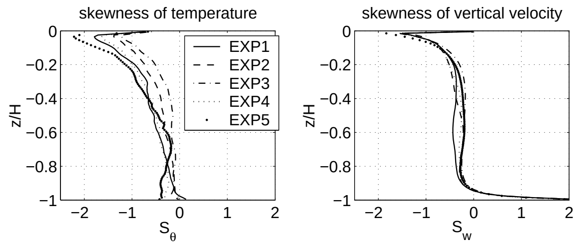

Figure 2. Skewness of vertical velocity S_w and temperature S_θ as a function of depth.

and suggest a generalization of the Millionshchikov hypothesis:

$$\langle w'^4\rangle = a_3\left(1+d_3\,S_w^2\right)\langle w'^2\rangle^2 \tag{2}$$
$$\langle\theta'^4\rangle = a_4\left(1+d_4\,S_{\theta'}^2\right)\langle\theta'^2\rangle^2 \tag{3}$$
$$\langle w'^3\theta'\rangle = a_5\left(1+d_5\,S_w^2\right)\langle w'^2\rangle\langle w'\theta'\rangle v \tag{4}$$
$$\langle w'\theta'^3\rangle = a_6\left(1+d_6\,S_\theta^2\right)\langle\theta'^2\rangle\langle w'\theta'\rangle \tag{5}$$
$$\begin{aligned}\langle w'^2\theta'^2\rangle = a_7\,&\big(2\,\langle w'\theta'\rangle^2+\langle w'^2\rangle\langle\theta'^2\rangle\\&+d_7\,S_wS_\theta\langle w'^2\rangle^{1/2}\langle\theta'^2\rangle^{1/2}\langle w'\theta'\rangle\big),\end{aligned} \tag{6}$$

with the skewnesses $S_w = \langle w'^3\rangle/\langle w'^2\rangle^{3/2}$ and $S_\theta = \langle\theta'^3\rangle/\langle\theta'^2\rangle^{3/2}$. The parameters a_i and d_i are found by postulating that in the limit of zero skewness formulae (2)–(6) should reduce to the Gaussian form (Eq. 1) and that in the limit of large skewness the turbulent regime is close to the top-hat regime [*Gryanik and Hartmann*, 2002]. *Gryanik et al.* [2005] determine them to be $a_i = 3$ and $d_i = 1/3$ for $i = 3, \ldots, 6$ and $a_7 = d_7 = 1$. With these parameters, *Gryanik et al.* [2005] call (2)–(6) their universal model. In the middle row of Fig. 1, the same fourth-order moments are plotted as in the top row, this time against their new parameterizations of the universal model that includes the effects of skewness.

The skewnesses S_w and S_θ for vertical velocity and temperature in Fig. 2 immediately reveal that the distribution of vertical velocity and temperature is essentially non-Gaussian in all experiments. Consequently, the Gaussian parameterization underestimates the fourth-order moments, in particular those which contain high powers of θ'. The improvements of the universal model over the Gaussian parameterization are particularly obvious where the skewness is large (Fig. 2), that is for $z/H > -0.3$. The moment $\langle w'^4 \rangle$ appears to be an exception. The Gaussian model represents this moment quite well. Contrary to all other moments it slightly overestimates $\langle w'^4 \rangle$ at mid-depth, but because S_w is small at mid-depth (Fig. 2), the universal model only slightly bends the curve further away from the diagonal. However, near the top of the computational domain where S_w is large, the Gaussian model does underestimate $\langle w'^4 \rangle$ (see inset figures in Fig. 1, which zoom to the range of 0.1 to 0.2). Here, the universal model improves the fit for near surface values (red markers), for which the skewness is large, but it only slightly changes the values near the bottom (blue markers) where the skewness is small.

The explained variance

$$\sigma_f^2 = 1 - \frac{\langle (y_i - f(x_i))^2 \rangle}{\langle (y_i - \langle y \rangle)^2 \rangle}$$

of the quasi-Gaussian parameterizations and *Gryanik et al.*'s universal model is compared in Fig. 3 for the different experiments. The explained variances for the quasi-Gaussian model range from 0.231 to 0.990. Including the effect of skewness by *Gryanik and Hartmann* [2002] and *Gryanik et al.* [2005] increases the explained variance to values ranging from 0.521 to 0.996. Only for $\langle w'^4 \rangle$, for which the Gaussian model is already good, can the universal model not explain more variance.

Finding the parameters a_i and d_i via a least-squares best-fit to all data (labeled "best fit" in Fig. 3) increases the explained variance of the generalized parameterization over the universal model even further (Fig. 3 and bottom row of Fig. 1). However, in particular for $\langle w'^4 \rangle$, $\langle w'\theta'^3 \rangle$, and $\langle \theta'^4 \rangle$ the universal model of *Gryanik et al.* [2005] is already nearly optimal. In most of the cases the global fit parameters are close to the values of the universal model (Fig. 1), except for $\langle w'\theta'^3 \rangle$.

As long as temperature and vertical velocity scale with θ_* and w_*, changing the surface heat flux and the Coriolis parameter over the ranges of 200 to 800 $\mathrm{W m^{-2}}$ and 0.7 to $1.4 \times 10^{-4}\,\mathrm{s^{-1}}$ does not alter the fit of the fourth-order moments to the universal model significantly. However, there appears to be a trend towards less explained variance with decreasing Coriolis parameter (not shown), which may serve as an indication that for weaker rotation the turnover times for water parcels become too short to be affected by rotation.

3. Conclusion

The model simulations presented in this note provide counterexamples for the Millionshchikov hypothesis. In all simulations the moments follow the general scaling laws for open-ocean convection with rotation. In this respect, the simulations and results of this note complement those of *Gryanik and Hartmann* [2002] and *Gryanik et al.* [2005] who consider shallow convection that is not rotationally controlled. Many more simulations with different numerical advection schemes, different values for implicit and explicit diffusivity for temperature, viscosity and hyper-viscosity are not shown here, but they all lead to the same conclusion that fourth-order moments should not be modeled based on the quasi-normality assumption. Instead, a model such as the universal model of *Gryanik and Hartmann* [2002] and *Gryanik et al.* [2005] that explicitly takes skewness into account appears to be more suited for fourth-order moments.

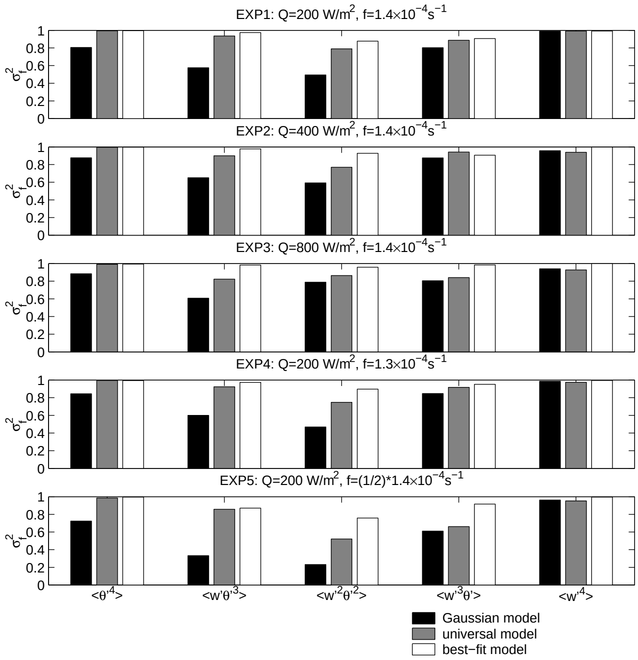

Figure 3. Explained variance of parameterizations for five experiments with different surface heat flux and Coriolis parameter. Except for $\langle w'^4 \rangle$, the universal model can explain more variance than the Gaussian model. In many cases, the universal model is nearly as good as the best-fit model for which the parameters have been found by a fit to data.

In the future, further parameterizations proposed by *Gryanik et al.* [2005] that include horizontal motion and salinity will be studied in a three-dimensional experiment.

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Martin Losch, Alfred-Wegener-Institut für Polar- und Meeresforschung, Postfach 120161, 27515 Bremerhaven, Germany. (e-mail: mlosch@awi-bremerhaven.de)