# 1 Introduction

The mean temperature of the Earth's surface has increased by approximately 0.8°K over the last century [18]. In view of this fact, a quantification of natural climate variations on different time scales is necessary to set the observed development in relation to long-term climate change. As instrumental data exist only for a small fraction of the younger history of the Earth (abundant data sets are available back to the mid-1850s [29]), indicators of climate, called *proxies*, are analyzed. Proxy data are gathered from natural recorders of climate variability. For instance, oxygen isotopes, exhibiting a temperature-dependent ratio, are built into the shells of foraminifera (marine organisms); information that is preserved in the sediments after the organisms die and sink to the ground. Together with other archives like ice cores, lake sediments, speleothems and tree rings, ocean sediments thus form the basis of paleoclimate analysis (the Greek word "paleo" standing for ancient or old). Whereas these archives were interpreted separately in the past, improved analysis techniques now allow for a systematic combination and a comparison of data sets as well as for their integration in numerical climate models.

To assess the extent of global climate interaction, statistical objectivity is needed to answer the question if climate variables at different locations significantly correlate. In this context, a central problem of the paleoclimate archives is the time uncertainty associated with the proxy data. Although crucial when proxy records are compared, this issue has not been sufficiently investigated yet.

The character and the degree of time uncertainty depend on the type of the climate archive and on the method used for establishing the time scale (termed *chronology* or *age model*). A simple example is found in the dating of ice cores, where the depth/age relationship is often determined by counting seasonally discernible annual layers. In this case, the ice core chronology gets distorted when a true layer is not detected or a false one is erroneously added. In order to avoid such measurement errors, investigators assign "uncertain layers" [1, p.3249] to layers that cannot be definitely classified. Some pragmatic statistics exist to account for the resulting inaccuracies in the age model (e.g., [1], [39], [40]).

An assessment of uncertainty is more complex for sediment cores. Ages are commonly estimated by translating measured concentrations of the radioactive carbon isotope $^{14}C$ into calendar ages. The transfer function is not deterministic, as the measurement itself and the applied reference curve (called *calibration curve*) are uncertain. Also, the number of points derived from radiocarbon dating is limited due to laboratory costs, so that linearity assumptions have to be made for the in between depth intervals. The problem of varying sedimentation rates thus adds up to the total age model uncertainty.

While issues dealing with the $^{14}C$ uncertainties are widely discussed in literature (e.g., [9], [15]), variations in the sedimentation process have been hardly considered until recently. One goal of this work is to quantify the magnitude of these variations. Therefore, the idea is to establish an adequate probabilistic representation of sedimentation, using the concept of stochastic processes. Mean and variance of the model shed light on the nature of the internal sedimentation variability. The influence of the variability on the precision of the chronology is evaluated by applying the model to real sediment core data.

Apart from the implications for the age model, a second goal is to gain general insights into the sedimentation process. Special interest is in testing the hypothesis of autocorrelated sedimentation rates [27], the latter being the amount of sediments deposited in one time unit. In this regard, sediment core data reveal whether local autocorrelation patterns exist. Also investigated is the dependence of the deposition process on changes in the climate state.

The structure of this works is as follows: In Section 2, the mathematical foundation is presented by introducing the methodologies of *Box-Jenkins* and *state space models*. Short descriptions are provided for the *Wiener process*, the *Brownian motion* and the *Brownian Bridge*. Section 3 contains information on the above outlined chronology type, a detailed literature review and the setup of the sedimentation model. In Section 4, an assemblage of 26 age models is analyzed, followed by a thorough discussion of the results. Some common technical terms used in climate science are explained in the attached glossary.

# 5 Conclusion

A time-discrete stochastic model for the sedimentation process has been outlined. Its deposition increments are made up of constant sedimentation rates and autoregressive (AR) realizations with Gaussian innovations. Embedding the model within a problem of dating paleoclimate archives, the main focus was on assessing the time uncertainty of the age control point (ACP) based age model, but also on gaining general insights into sedimentation characteristics.

Therefore, two cases were examined: an age interval with one depth/age pair, leading to the *unconstrained process*, and an age interval framed by two depth/age pairs at the ends, termed the *constrained process*. While the first setting exhibits increasing magnitudes of depth variations away from the age control point, the second shows a Brownian bridge like shape of variability. Analytical expressions for expected mean and variance with respect to depth have been derived for either setup.

Two parameters $\phi$ and $J$ control the variations in the stochastic process and the magnitude of time uncertainty, the former indicating the degree of autocorrelation. The latter, called jitter, measures the ratio of variability and mean sedimentation rate. Its definition was based on the assumption that sites with high mean sedimentation rates also show high fluctuation in the amount of sediments deposited per time step. Taking ACP data as realizations of an integrated autoregressive process (formulated as a Box-Jenkins ARIMA(1,1,0) model), a parameter estimation concept using results from state space methodology was devised. The Kalman filter and a diffuse prior density were employed to compute Maximum Likelihood estimates.

Uncertainties in the age control points have not been considered in the definition of the model.

After having established the theory, the model was applied to an assemblage of 26 chronologies for 21 sediment cores with high spatial coverage, spanning on average about 32kyr (one age model that starts at 0.1 and ends at 1028kyr BP excluded). The numbers of age control points range from 12 to 4926. Three of the 26 age models that violate the model assumptions (revealing differing magnitudes of variability within the depth profile) were excluded prior to the experiments. Analysis of the remaining 23 chronologies was performed with regard to a wide range of questions:

a) Do sedimentation rates show an autoregressive pattern?

Concerning the memory of sedimentation rates, 15 out of 23 age models yielded AR coefficients $\phi$ that are significant at a 5% level. Thus, two-thirds of the chronologies support the hypothesis of autocorrelated sedimentation rates by Huybers and Wunsch [27], leading to larger time uncertainties in

ACP based age models. The autocorrelation in the sedimentation rate might be explained by one main driver of sedimentation, the biological productivity. This productivity is climate-dependent, and most climate variables are autocorrelated.

b) Is ARIMA(1,1,0) an adequate tool for modeling accumulation histories?

As far as the data permit validation, the ARIMA(1,1,0) model was found to give an adequate representation of the sedimentation process. Apart from the lack of records with high temporal resolution, problems were detected when deterministic phenomena such as sediment compaction and climate conditions influenced the deposition behavior. These were determined by contrasting the ACP sets with wet bulk density (interpreted as a measure of compaction) and $\delta^{18}O$ records (taken as a temperature proxy) in some exemplary cases. An extension of the model allowing for slow variations in the sedimentation rates however led to estimates that are only slightly biased towards larger depth variations/time uncertainties.

c) How variable is the sedimentation process itself? How uncertain are ACP dated age models?

The internal sedimentation variability was quantified for a 10kyr interval with age control points at both ends (supposed to be common for many sediment cores). With the respective parameter estimates, it was seen that the $1\sigma$ time uncertainties amount to approximately 1kyr on average at the interval midpoint. One age model from the Arabian Sea even suggested a $1\sigma$ deviation of 3.25kyr. For the applied 10kyr setting, the magnitude of internal variations exceeded the ACP uncertainties available for the age models. In terms of a precise time scale, this means that more age control points should be taken when AR coefficient and jitter are large, whereas 10kyr ACP intervals seem to be sufficient for small values of $\phi$ and $J$.

d) Does a spatial pattern exist in terms of autocorrelated sedimentation rates and sedimentation variability?

Common characteristics in spatial terms exist for $\phi$ as well as for $J$ values. Positively autocorrelated sedimentation rates were found for cores from the Cariaco Basin and the West Pacific region, while negative AR coefficients apparently dominate the Norwegian Sea. However, four out of five negative $\phi$ estimates from the Norwegian Sea are not significant according to the statistical test. The spatial distribution of the jitter, whose heuristic definition was confirmed by experimental comparison of mean sedimentation rates with observed variations, suggests a similar pattern. Consistent jitter values around 0.73 were obtained for the Cariaco Basin and the West Pacific. Thus, magnitudes of innovations and mean sedimentation rates approximately coincide in these regions. For Norwegian Sea sites, estimates that range from 0.87 to 36.72 give rise to the hypothesis that extreme oscillations characterize the sedimentation process. Age models from the Arabian Sea display contradicting results in terms of both $\phi$ and $J$ estimates.

e) Are the sedimentation rate and the variability sensitive to the climate state?

The parameter estimates could not be classified into time windows with a distinct climate state (e.g., glacial/interglacial). One reason was in the limited number of analyzed cores. However, $\delta^{18}O$ records were used for correlation with sedimentation rate profiles in five cases, revealing that four accumulation histories are related to climate changes displayed by $\delta^{18}O$. This means that larger uncertainties in the age model are expected for records that cover one or more climate transitions. The correlation values do not suggest a consistent type of sensitivity, though: Two of them are greater and two smaller than zero.

This thesis gave first insights into the time uncertainty in sediment cores caused by the sedimentation rate variability. It showed that most sedimentation rates are autocorrelated in time and proposed that the autocorrelation and the relative sedimentation rate variability have spatial coherent patterns. Therefore, the obtained sedimentation parameter estimates might be taken as indicators for the time uncertainty in sediment cores that do not have enough age control points to reflect sedimentation characteristics. Case studies demonstrated that climate proxies ($\delta^{18}O$) and sedimentation rates derived from the same core are often related. This suggests that time periods that are particularly interesting to climate research (as they record strong variations) are also very time uncertain.

In my work, sedimentation records were analyzed from a mathematical perspective. The mechanisms causing the sedimentation variability were not discussed or investigated. In the future, cooperation with geologists and the bio-geo department is needed. Geologists might provide essential input about the nature of sedimentation variability, the compaction of sediment cores and artifacts in the depth/age relationship due to the mechanical stress on the cores during recovery. Modeling the sedimentation flux by using ocean models (including biogeochemistry) can show how climate, biological productivity and sedimentation rates are connected. The knowledge about the drivers of the sedimentation variability has to be complemented by data analysis and an improvement of the methodology.

A necessary next step is to integrate ACP time uncertainties in the stochastic model. This will affect the parameter estimation as well as estimates of the overall time uncertainty. The numerical method of the parameter estimation has to be improved to cope with a larger proportion of missing observations. While the results of this work clearly indicate that sedimentation rates are autocorrelated, long-memory processes could be an even better description of the sedimentation process and should therefore be tested. Finally, the analysis should be extended to a larger array of chronologies to get a more robust picture of the spatial pattern and the climate dependence of sedimentation variability.

In the systematic analysis of paleoclimate data, understanding and quantifying time uncertainty is crucial to infer on the past climate state. My work hopefully made a contribution to this issue.