Polarforschung 59/3: 113–128, 1989 (erschienen 1991)

Parameterization of Melt Rate and Surface Temperature on the Greenland Ice Sheet

By Niels Reeh*

Summary: Melt rate and surface temperature on the Greenland ice sheet are parameterized in terms of snow accumulation, mean annual air temperature and mean July air temperature. Melt rates are calculated using positive degree-days, and firn warming (i.e. the positive deviation of the temperature at 10-15 m depth from the mean annual air temperature) is estimated from the calculated amount of refrozen melt water in the firn.
A comparison between observed and calculated melt rates shows that the parameterization provides a reasonable estimate of the present ablation rates in West Greenland between 61° N and 76° N. The average equilibrium line elevation is estimated to be about 1150 m and 1000 m for West and East Greenland respectively, which is several hundred metre lower than previous estimates. However, the total annual ablation from the ice sheet is found to be about 280 km³ of water per year which agrees well with most other estimates. The melt-rate model predicts significant melting and consequently significant firn warming even at the highest elevations of the South Greenland ice sheet, whereas a large region of central Greenland north of 70° N experiences little or no summer melting. This agrees with the distribution of the dry snow facies as given by BENSON (1962).

Zusammenfassung: Für das grönländische Inlandeis werden die Schmelzraten und oberflächennahen Firntemperaturen mit Hilfe von Schneeakkumulation sowie Jahresmittel und Julimittel der Lufttemperatur parameterisiert. Zur Berechnung der Schmelzraten werden positive Gradtage benützt, und die Erwärmung des Firns (d.h. die positive Abweichung der Temperatur der Schneedecke in einer Tiefe von 10-15 m vom Jahresmittel der Lufttemperatur) wird aus der berechneten Menge wiedergefrorenen Schmelzwassers abgeschätzt. Ein Vergleich zwischen beobachteten und berechneten Schmelzraten zeigt, daß die Parameterisierung eine vernünftige Abschätzung für die gegenwärtigen Schmelzraten in Westgrönland, zwischen 61° und 76° nördlicher Breite, bietet. Die berechnete mittlere Höhe der Gleichgewichtslinie ist ca. 1150 m für Westgrönland und ca. 1000 m für Ostgrönland und liegt damit einige hundert Meter niedriger als frühere Berechnungen ergaben. Der Gesamtbetrag der mittleren jährlichen Ablation beläuft sich jedoch auf 280 km³ Wasseräquivalent pro Jahr und stimmt somit gut mit den meisten anderen Abschätzungen überein. Das Modell zur Berechnung der Schmelzraten ergibt für das südgrönländische Inlandeis, sogar für die höchsten Erhebungen, erhebliche Abschmelzraten und damit auch Erwärmung der Firndecke, während weite Bereiche des zentralen Inlandeises nördlich des 70. Breitengrades wenig oder keine sommerliche Abschmelzung erfahren. Dies steht in Übereinstimmung mit der Verteilung der Trockenschneezone, wie sie von BENSON (1962) angegeben wird.

1. INTRODUCTION

Models of the dynamics and thermodynamics of ice sheets and glaciers depend on the boundary conditions at the ice-sheet surface involving mass balance and surface temperature. The surface mass balance (the net effect of annual snow-accumulation rate and annual melt rate) and the surface temperature (the temperature at 10-15 metre depth where annual temperature variations can be neglected) result from quite complex processes, involving the general atmospheric circulation pattern and the energy balance at the ice-sheet surface. These processes are not yet so well understood that precise distributions of mass balance and surface temperature can be determined by model calculations. This is even more true, when past or future variations are considered. Moreover, the most sophisticated of the time-dependent ice-dynamic models now available are so complex that coupling with general circulation and energy balance models would result in very long computation times.

These facts justify a simpler approach, e.g. parameterizing the surface mass balance and temperature in terms of a few climatic parameters as for example mean-annual air temperature and summer air temperature. Past and future changes in the boundary conditions can then be calculated by letting perturbations in these climatic parameters control the variations in surface mass balance and temperature. The success of this approach depends on the accuracy of the parameterizations which, of course, must provide realistic pictures of the actual distribution of mass balance and temperature over the ice sheet surface.

This study reports on parameterizations of melt-rates and surface temperatures on the Greenland ice sheet. The parameterizations were meant to be used as boundary conditions for ice-dynamic model studies of the Greenland ice sheet under different climatic conditions, (LETRÉGUILLY et al. in press). The climatic 'forcings' for these studies were air-temperature and precipitation variations, and the annual melt rates and surface-temperatures, therefore, had to be expressed in terms of these variables.

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Manuscript received 27. 07. 1990; accepted 04. 01. 1991

The parameterization problem is solved in four steps:

1) Mean annual air temperatures (TMA) and mean July air temperatures (TMJ) over the ice sheet are expressed in terms of latitude and elevation based on the study by OHMURA (1987).
2) Positive degree-days are calculated from TMA, TMJ, and a stochastic term (TR) accounting for temperature deviations from the regular, long-term annual cycle, and also accounting for daily temperature variations, which, although they contain a large deterministic component, can be approximately accounted for by a stochastic term.
3) Snow and ice melt are calculated from the positive degree-days, using different degree-day factors for snow and ice melt. Refreezing of percolating meltwater in the snow or firn pack is considered.
4) The surface temperature (TS) is calculated by means of the mean annual air temperature and the annual melt rate.

The approach showed up to yield very reasonable results. Step 4 was accomplished by using all available surface temperature data on the ice sheet to deduce a simple empirical relationship between the „firn warming" (the difference between surface and mean-annual air temperatures) and the annual melt rate.

2. AIR TEMPERATURE

Ohmura (1987) used all available air temperature data from Greenland to construct mean annual and mean monthly air-temperature maps. In contrast to a fairly large number of observations of snow/ice surface temperatures - about 170 stations distributed over the ice sheet (MOCK & WEEKS 1965, SCHYTT 1955, NOBLES 1960, MÜLLER et al. 1977, RADOK et al. 1982, CLAUSEN et al. 1988) - the number of stations with air-temperature records are rather limited. OHMURA (1987) lists 48 stations, 26 of which are positioned on the coast or in other ice-free areas, leaving only 22 ice sheet stations with air-temperature records. At the majority of these stations, the temperature observations cover a few summer months, only. Leaving out the records from Dye2 and Dye3 which are problematic for the reasons discussed by OHMURA (1987), five stations, only, remain with air-temperature records that cover the full annual cycle. The length of these records varies from 1 to 3 years, but all records have been referred to the period 1951-1960, (OHMURA 1987).

In spite of the limited data, the parameterizations to be presented in this work are based on the records from the ice sheet stations only. The reason for leaving out the coastal and other non-ice-sheet stations is that the climate of the coastal areas is different from the climate near and on the ice sheet margin. Particularly in the summer months temperatures are generally warmer at the ice margin than on the coast. This „inland effect" is e.g. illustrated by data published by HOLZAPFEL et al. (1939, p. 135) who compare temperatures observed in 1930/31 at Ummanak, West Greenland (a coastal station) and at Kamarujuk (Qaumarujuk, a station at the ice sheet margin about 50 km farther inland). Both stations are close to sea level. The mean annual, respectively mean summer (June, July, August) temperatures were 0.9 K respectively 2.6-3 K warmer at Qaumarujuk than at Ummanaq.

Mean annual air temperature

A linear regression of ice sheet mean annual air temperatures (TMA) on elevation E (m) and Latitude L (°N) yields the following equation

$$\text{TMA} = 48.38 - 0.007924\,E - 0.7512\,L \tag{1}$$

The multiple regression coefficient is 0.995 and the rms (root-mean-square) value of the residuals is 0.71 K. The temperature data from the ice sheet are too scanty to justify a differentiation between East and West Greenland as done by OHMURA (1987), who included also data from the ice-free areas. However, the inversion occurring at low altitudes in North Greenland (SCHYTT 1955, MOCK & WEEKS 1965, OHMURA 1987, see also Fig. 7) is accounted for by letting the temperature-elevation gradient below 300 m change linearly from -0.007924 K/m at 70° N to zero at 75° N, and keeping the gradient below 300 m at zero north of 75° N.

In Table 1 observed and estimated mean annual air temperatures for all Greenland stations are compared. The rms-value of the residuals is 1.2 K, which is significantly higher than the rms-value of the residuals for the ice sheet stations (0.7 K). The mean value of the residuals for the stations in the ice-free land (mainly coastal stations)

STATION	LONG (°W)	LAT (°N)	ELEV (m)	TMA_O (°C)	TMA_E (°C)	RES (K)
Ice-free area						
QANAQ	69.20	77.48	15	-10.80	-12.20	-1.40
CAREY ØER	72.92	76.73	10	-9.80	-11.64	-1.84
DUNDAS	68.80	76.57	21	-10.90	-11.51	-0.61
THULE AFB.	68.83	76.52	11	-11.50	-11.48	0.02
TUTO WEST	68.67	76.47	250	-11.30	-11.44	-0.14
TUTO I	68.23	76.42	486	-12.50	-12.88	-0.38
CAPE ATHOL	69.37	76.32	10	-9.20	-11.33	-2.13
PEARY LODGE	56.22	74.32	30	-9.10	-9.53	-0.43
UPERNAVIK	56.17	72.78	63	-7.30	-7.84	-0.54
UMANAK	52.00	70.67	40	-4.80	-5.30	-0.50
JAKOBSHAVN	51.05	69.22	40	-4.20	-3.93	0.27
GODHAVN	53.52	69.23	25	-3.70	-3.83	-0.13
EGEDESMINDE	52.75	68.70	47	-4.60	-3.60	1.00
SØNDERSTRØM	50.80	67.02	55	-4.70	-2.40	2.30
HOLSTEINSBORG	53.67	66.92	9	-3.30	-1.96	1.34
SUKKERTOPPEN	52.87	66.40	24	-1.00	-1.69	-0.69
GODTHÅB	51.90	64.17	27	-0.70	-0.04	0.66
FÆRINGEHAVN	51.55	63.70	7	-0.30	0.47	0.77
FREDERIKSHÅB	49.72	62.00	16	0.20	1.68	1.48
GRØNNEDAL	48.50	61.50	0	0.50	2.18	1.68
IVIGTUT	48.17	61.20	30	1.80	2.17	0.37
NARSSARSSUAQ	45.42	61.18	26	2.30	2.21	-0.09
JULIANEHÅB	46.05	60.72	34	1.70	2.50	0.80
NARSSAQ	45.97	60.90	31	1.60	2.39	0.79
KAP MORRIS J.	33.37	83.63	4	-18.80	-16.82	1.98
JØRGEN BR. FJ.	30.50	82.17	5	-15.00	-15.72	-0.72
NORD	16.67	81.60	35	-16.50	-15.30	1.20
BRITANIA SØ	23.60	77.15	229	-10.20	-11.95	-1.75
DANMARKSHAVN	18.77	76.77	12	-11.80	-11.66	0.14
DANEBORG	20.22	74.30	13	-10.00	-9.49	0.51
MESTERS VIG	23.90	72.25	10	-9.70	-7.01	2.69
KAP TOBIN	21.97	70.42	41	-7.40	-5.01	2.39
APUTITEQ	32.30	67.78	19	-4.10	-2.69	1.41
ANGMAGSSALIK	37.57	65.60	35	-0.80	-1.18	-0.38
TINGMIARMIUT	42.13	62.53	10	-0.70	1.33	2.03
PRINS CHRIST. S.	43.12	60.03	76	1.20	2.68	1.48
Ice sheet						
NORTHICE	38.48	78.07	2343	-30.30	-28.83	1.47
CAMP CENTURY	61.08	77.18	1871	-24.10	-24.43	-0.33
SITE 2	56.08	77.00	1914	-23.80	-24.63	-0.83
TUTO EAST	67,92	76.38	801	-15.10	-15.35	-0.25
WESTSTATION	51.12	71.18	954	-12.60	-12.65	-0.05
ST. CENTRALE	40.63	70.92	2961	-28.50	-28.36	0.14

Tab 1: Comparison of observed (TMA_O) and estimated (TMA_E) mean annual air temperatures in Greenland.

Tab. 1: Vergleich von beobachtetem (TMA_O) und berechnetem (TMA_E) Jahresmittel der Lufttemperatur von Grönland.

is +0.53 K, illustrating that the regression equation based on temperature data from the ice sheet stations, in general over-estimates the mean annual air temperatures on the coast, thus confirming the „inland effect" mentioned previously.

Mean July air temperature

Since a linear regression does not yield an adequate representation of the observed mean July air temperatures (TMJ) for the ice sheet stations, a slightly more complicated parameterization is chosen. Air-temperature observations in West Greenland around 76.5° N and 70° N define the elevation of the July zero-degree isotherm to be

$$E_0 = 5960 - 66\,L$$

Below this elevation the temperature-elevation gradient is taken as -0.0066 K/m. Above the zero-degree isotherm the gradient is taken to vary linearly from -0.007 K/m at 84° N to -0.0064 K/m at 64° N. The rms-value of the ice-sheet-station residuals from this parameterization is 0.92 K, which is slightly less than the corresponding rms-value obtained with a linear regression model. The mean value of the residuals is -0.01 K, showing that, on the average, the parameterization provides correct estimates of the observed July air temperatures on the ice sheet.

In Table 2, observed and estimated mean July air temperatures for all Greenland stations are compared. The large positive residuals for many coastal stations suggest that the „inland effect" is generally very significant in the summer period. The suggested parameterizations of mean annual and mean July air temperatures are in good agreement with the diagrams presented by OHMURA (1987).

3. THE DEGREE-DAY MODEL

The annual melt rate actually depends on the energy balance at the ice sheet surface. However, when dealing with complex 3D ice sheet modelling, it is not feasible to make detailed energy-balance calculations for all points on the ice sheet surface at all times when melt rates are needed in the calculations. A simpler approach, using air temperatures as the factor that determines the melt rate, seems to be a more practicable approach.

Various relationships have been suggested between air temperature (annual mean temperature or summer mean temperature) and annual melt rates in the form of second or third degree polynomials (OERLEMANS & Van der VEEN 1984, p. 185, KRENKE 1975). However, an approach that uses mean summer temperature or mean annual temperature as the only parameter to determine the melt rate, can neither account for the effect of snow accumulation nor for the length of the meltseason. Moreover, extrapolation with a second- or third order equation may lead to erroneous results. For these reasons, a melt-rate model based on a degree-day approach is preferable. A further argument in favour of a degree-day approach is provided by the work of BRAITHWAITE & OLESEN (1984, 1989) documenting a high correlation between positive degree-days and melt rates at West Greenland ice-margin locations.

The degree-day model is illustrated in Fig. 1. The annual temperature cycle is supposed to follow a cosine function

(2) $$TCA = TMA + (TMJ-TMA) \cos(2 \pi t/A)$$

where TMA and TMJ are the long-term mean annual and mean July air temperatures, respectively. t is time and A = one year. It is, of course, an approximation to describe the annual temperature cycle by means of a single cosine function. However, considering the other approximations introduced in the degree-day model, it is hardly justified to use a more sophisticated expression for the average annual temperature cycle.

A more serious problem is that, if TMJ < 0° C, then the positive degree days (PDD) as determined by means of Equation (2) becomes zero. This is not the case in real world. Even if the average temperature of the warmest summer month is below the freezing point, there is likely to be days when the temperature exceeds the zero-degree mark. For example, a daily temperature cycle with an amplitude of typically 5 K will cause a positive degree-day contribution if TMJ > -5° C. Also random temperature deviations from the average annual cycle are likely to cause positive temperatures in the spring or in the fall (may be even in the winter) although the average temperature in these seasons may be well below the freezing point. Summing up, the deviations of the actual temperature from the long term average cycle given by Equation (2) is composed of a high-frequency term which is mainly deterministic (the daily temperature cycle) and a stochastic term. As an approximation, the combined effect of these terms is in the model accounted for by a statistic (TR), which is normally distributed, centered on the curve given by Equation (2), and having a standard deviation s, see Fig. 1. This approach follows the approach suggested by BRAITHWAITE (1984).

The probability for having a temperature in the small interval dT centered at T at time t is therefore

$$p = 1/(s \sqrt{2\pi}) \exp[-(T-TCA)^2/(2s)^2] \, dT$$

STATION	LONG. (°W)	LAT. (°N)	ELEV. (m)	TMJ_O (°C)	TMJ_E (°C)	RES (K)
Ice-free area						
QANAQ	69.20	77.48	15	5.60	5.50	-0.10
NORTH ICS. I	66.97	76.92	630	2.00	1.69	-0.31
CAREY ØER	72.92	76.73	10	4.50	5.86	1.36
DUNDAS	68.80	76.57	21	4.90	5.86	0.96
THULE AFB.	68.83	76.52	11	5.70	5.95	0.25
TUTO WEST	68.67	76.47	250	6.50	4.39	-2.11
TUTO I	68.23	76.42	486	3.80	2.86	-0.94
CAPE ATHOL	69.37	76.32	10	6.40	6.04	-0.36
PEARY LODGE	56.22	74.32	30	6.10	6.78	0.68
UPERNAVIK	56.17	72.78	63	6.10	7.23	1.13
UMANAK	52.00	70.67	40	8.50	8.31	-0.19
JAKOBSHAVN	51.05	69.22	40	8.70	8.94	0.24
GODHAVN	53.52	69.23	25	7.60	9.03	1.43
EGEDESMINDE	52.75	68.70	47	6.40	9.12	2.72
SØNDERSTRØM	50.80	67.02	55	10.50	9.80	-0.70
HOLSTEINSBORG	53.67	66.92	9	6.60	10.14	3.54
SUKKERTOPPEN	52.87	66.40	24	8.20	10.27	2.07
GODTHÅB	51.90	64.17	27	7.10	11.22	4.12
FÆRINGEHAVN	51.55	63.70	7	6.00	11.56	5.56
FREDERIKSHÅB	49.72	62.00	16	6.10	12.24	6.14
NARSSARSSUAQ	45.42	61.18	26	11.20	12.53	1.33
JULIANEHÅB	46.05	60.72	34	7.50	12.68	5.18
NARSSAQ	45.97	60.90	31	7.90	12.62	4.72
KAP MORRIS J.	33.37	83.63	4	1.40	2.89	1.49
JØRGEN BR. FJ.	30.50	82.17	5	6.20	3.53	-2.67
KAP HA. MOLTKE	29.95	82.17	13	5.40	3.47	-1.93
BRITANIA SØ	23.60	77.15	229	7.30	4.23	-3.07
DANMARKSHAVN	18.77	76.77	12	3.50	5.83	2.33
DANEBORG	20.22	74.30	13	3.60	6.90	3.30
MESTERS VIG	23.90	72.25	10	6.00	7.81	1.81
KAP TOBIN	21.97	70.42	41	2.50	8.41	5.91
APUTITEQ	32.30	67.78	19	2.40	9.70	7.30
ANGMAGSSALIK	37.57	65.60	35	6.60	10.55	3.95
TINGMIARMIUT	42.13	62.53	10	5.40	12.05	6.65
PRINS CHRIST. S.	43.12	60.03	76	7.00	12.70	5.70
IVIGTUT	48.17	61.20	30	9.30	12.50	3.20
Ice sheet						
NORTHICE	38.48	78.07	2343	-9.90	-10.46	-0.56
SIERRA	62.33	77.23	1719	-4.80	-5.80	-1.00
CAMP CENTURY	61.08	77.18	1871	-6.40	-6.81	-0.41
SITE 2	56.08	77.00	1914	-7.00	-7.02	-0.02
NORTH ICS. III	66.98	76.93	700	1.40	1.22	-0.18
NORTH ICS. II	66.97	76.92	650	2.20	1.56	-0.64
TUTO EAST	67.92	76.38	801	0.80	0.79	-0.01
CAMP WATKINS	47.50	74.67	2659	-9.90	-10.92	-1.02
JARL JOSET	33.35	71.47	2867	-11.90	-10.74	1.16
WESTSTATION	51.12	71.18	954	0.80	2.05	1.25
ST. CENTRALE	40.63	70.92	2961	-12.80	-11.09	1.71
HIRAN 28	36.17	70.62	3139	-12.30	-12.02	0.18
CAMP IV EGIG	49.63	69.67	1004	1.40	2.38	0.98
HIRAN 30	43.17	69.55	2558	-7.70	-7.79	-0.09
HIRAN 27	35.92	69.38	2755	-7.30	-9.00	-1.70
HIRAN 26	36.50	68.25	2925	-8.40	-9.58	-1.18
HIRAN 29	42.33	68.07	2593	-7.70	-7.32	0.38
MINT JULEP	47.77	66.28	1829	-2.60	-1.56	1.04

Tab. 2: Comparison of observed (TMJ_O) and estimated (TMJ_E) mean July air temperatures in Greenland.

Tab. 2: Vergleich von beobachtetem (TMJ_O) und berechnetem (TMJ_E) Julimittel der Lufttemperatur von Grönland.

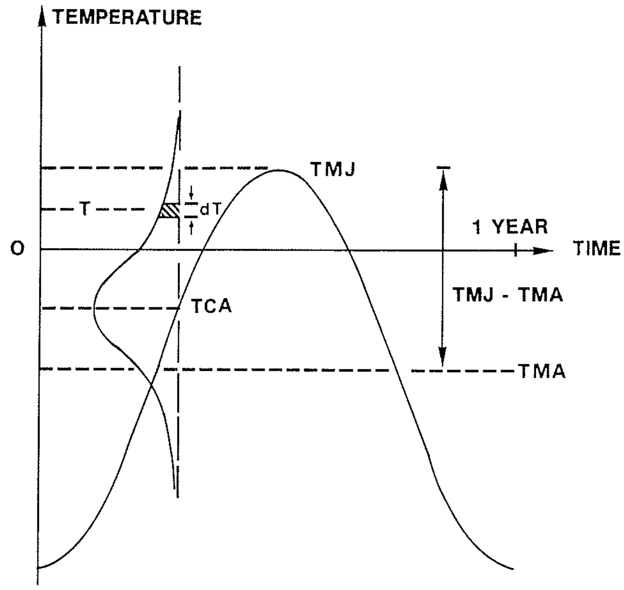

Fig. 1: Temperature variation used to calculate the number of positive degree days per year (PDD). For further explanation see text.

Abb. 1: Temperaturschwankung, die zur Berechnung der Anzahl positiver Gradtage (PDD) verwendet wurde. Weitere Erläuterung siehe Text.

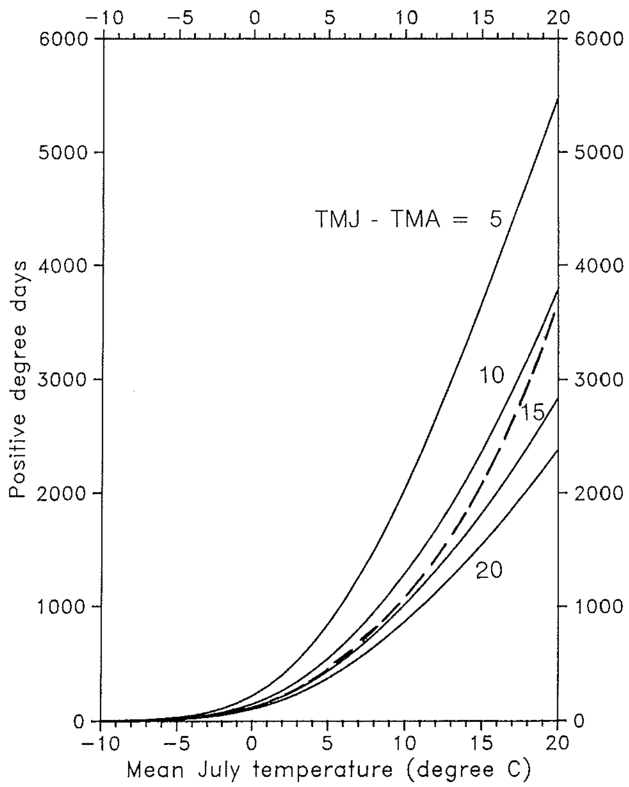

Fig. 2: Positive degree days as calculated from Equation (4) as a function of mean July temperature for various values of the amplitude of the annual temperature cycle TMJ-TMA. s has been set equal to 4.5 K. The curve shown by a dashed line is based on an equation by KRENKE (1975), see text.

Abb. 2: Anzahl positiver Gradtage als Funktion des Julimittels der Lufttemperatur, berechnet nach Gleichung (4), für unterschiedliche Amplituden des Jahrestemperaturzyklus TMJ-TMA. s wurde gleich 4,5 K gesetzt. Die gestrichelt gezeichnete Kurve entspricht einer Gleichung von KRENKE (1975), siehe Text.

where TCA is given as a function of time by means of Equation (2).

The corresponding degree-day contribution is

(3) $$\mathrm{Tdt}\ p = T\ dt/(s\sqrt{2\pi})\ \exp[-(T-TCA)^2/(2s)^2]\ dT$$

The annual degree-day contribution from the temperature interval dT centered at T is found by integrating Equation (3) with respect to time t over one annual cycle

$$d(PDD) = \int_0^A \{T/(s\sqrt{2\pi})\ \exp[-(T-TCA)^2/(2\ s)^2]\ dT\}\ dt$$

and the positive degree days in one year can therefore be calculated as

(4) $$PDD = \int_0^\infty T\ \{\int_0^A 1/(s\sqrt{2\pi})\ \exp[-(T-TCA)^2/(2s)^2]\ dt\}\ dT$$

where TCA is given by Equation (2).

In Fig. 2 positive degree-days, as calculated from Equation (4) using numerical integration, are displayed as a function of mean July temperature for various values of the amplitude of the annual temperature cycle. In the calculations s was set equal to 4.5 K.

Fig. 2 shows that positive degree-days depend strongly on July temperature, and that the amplitude of the annual temperature cycle is also important. This clearly indicates that melt-rate parameterizations based on a single temperature parameter, are too simple. As an illustration, the relationship between summer temperature (TSU) and melt rate (MR) suggested by KRENKE (1975):

$$MR = (9.5 + TSU)^3/1000$$

has been converted to a relationship between summer temperature and PDD by means of the factor 0.007 m of melt/PDD (see the following section), and is plotted in Fig. 2. The curve lies between the two curves calculated by means of Equation (4) corresponding to annual temperature amplitudes of 10 and 15 K, respectively.

4. THE SNOW-AND-ICE-MELT MODEL

The snow-and-ice-melt model is essentially similar to the model described by BRAITHWAITE & THOMSEN (1984), except that rainfall is neglected, i.e. precipitation is assumed to occur as snowfall, only.

The available positive degree-days (PDD) as calculated by means of Equation (4), are used to melt snow and ice in the following order:

1) Snow (if present) is melted. The meltwater is supposed to percolate into the snowcover and refreeze as superimposed ice. Runoff does not occur untill the amount of superimposed ice exceeds a given fraction (PMAX) of the snow cover.
2) The superimposed ice is melted.
3) Glacier ice is melted.

The process may stop at any of the stages 1 to 3 depending on the melt potential available, i.e. the magnitude of the PDD. Degree-day factors for snow and ice melt are set to 0.003 and 0.007 m of water per degree-day (BRAITHWAITE & OLESEN 1989). The low degree-day factor for snow melt is introduced to account for the generally higher albedo of a snow surface compared with the albedo of an ice surface.

5. MASS BALANCE ELEVATION RELATIONSHIP FOR THE ICE MARGIN

The air-temperature parameterizations, the degree-day model, and the snow-and-ice-melt model are now combined in order to estimate the mass balance elevation relationship for the marginal areas of the ice sheet. The input to this model is latitude, annual snow accumulation as a function of elevation, and the parameters of the snow-and-ice-melt model, i.e. degree-day factors of 0.003 m of water/degree day and 0.007 m of water/degree-day for snow and ice melt, respectively, and the factor PMAX = 0.6 limiting the formation of superimposed ice. As previously mentioned, the degree-day factors are chosen in accordance with the values suggested by BRAITHWAITE & OLESEN (1989), whereas the choice of PMAX is rather arbitrary. However, choosing PMAX = 0.6 results in an estimate of the amount of melt from the total ice sheet which agrees well with other estimates, see discussion in a later section.

In Fig. 3, calculated and observed mass-balance elevation relationships at four locations in West Greenland are compared. The locations are Nordbogletscher, 61.5° N (CLEMENT 1983), Qamanârssûp sermia, 64.5 °N (BRAITHWAITE 1983), Pâkitsoq, 69.5° N (THOMSEN 1987), and Nunatarssuaq ice ramp, 77° N (NOBLES 1966). The annual snow accumulation as a function of elevation is taken from the precipitation map shown in Fig. 5 (compiled by OHMURA & REEH in press). This means, that precipitation on the ice sheet margin is assumed to be in the form of snow, only. This, of course, is an approximation since some of the summer precipitation will fall as rain. Only a fraction of the rain will refreeze at the surface and contribute to the accumulation.

Strictly speaking, the calculated annual balances correspond to the average temperature conditions in the period 1951-1960, since all air-temperature observations have been referred to that period (OHMURA 1987). On the other hand, the observations represent atmost a few years of annual balances at different times. Keeping this in mind, the agreement between calculated and observed mass balances is satisfactory. In fact, it is surprising that a model with so few parameters can do so well.

The influence of local accumulation-rate variations on the net balance is illustrated in Fig. 4 in which the calculated net balance for Tuto ramp in the Thule area, northwest Greenland is compared to the observed net balance for the balance year 1953/54 (SCHYTT 1955). In this calculation, the observed winter balance on the Tuto ramp was used to represent the snow accumulation. Again, the agreement between calculated and observed net balances is acceptable. The figure illustrates the large influence of snow accumulation on the net balance. The increase of the winter balance with decreasing elevation (probably caused by wind drift) causes reduced melting near the ice margin at 500 m elevation. Consequently, the minimum net balance is not found at the margin but at a 50-70 m higher elevation.

Fig. 6 displays the calculated net balance as a function of elevation and latitude for the ablation zone of the Greenland ice sheet. The equilibrium line ELA (zero mass balance), the snow line SLA (separating the superimposed ice zone from the wet snow zone), and the runoff line RLA (above which all meltwater refreezes in the snow pack) are also shown in the figure.

There appears to be large differences between the western and eastern slopes of the ice sheet, mainly due to large differences in snow accumulation. In South Greenland, a much larger snow fall on the eastern slope compared to that on the western slope causes a comparatively higher net balance on the eastern slope. This is reflected in the relatively low ELA, SLA, and RLA for Southeast Greenland. North of about 70° N conditions are reversed. At these latitudes, less snow accumulation in East Greenland as compared to West Greenland, causes a higher elevation of the equilibrium, snow and runoff lines than in West Greenland. In fact, the highest ELA in East Greenland is calculated to be about 1300 m at 72-73° N. In West Greenland the highest ELA is calculated to be 1550 m at 66-67° N, i.e. somewhat higher and much farther to the south. The undulations on the ELA, SLA, and RLA curves are also due to accumulation rate effects. Maxima, respectively minima of the curves correlate with minima, respectively maxima of the accumulation-rate distribution, as can be seen by comparison with the accumulation-rate map shown in Fig. 5.

The average ELA is calculated to be about 1150 m and 1000 m for West and East Greenland, respectively, i.e. several hundred metre lower than the previous estimates of about 1500 m (BENSON 1962, OHMURA et al. 1986). The low ELA estimate of the present work, however, is supported by observations, see Fig. 3. The low average ELA for East Greenland in respect to that for West Greenland is to a large extent due the depression of the ELA in Southeast Greenland caused by the large snow accumulation there.

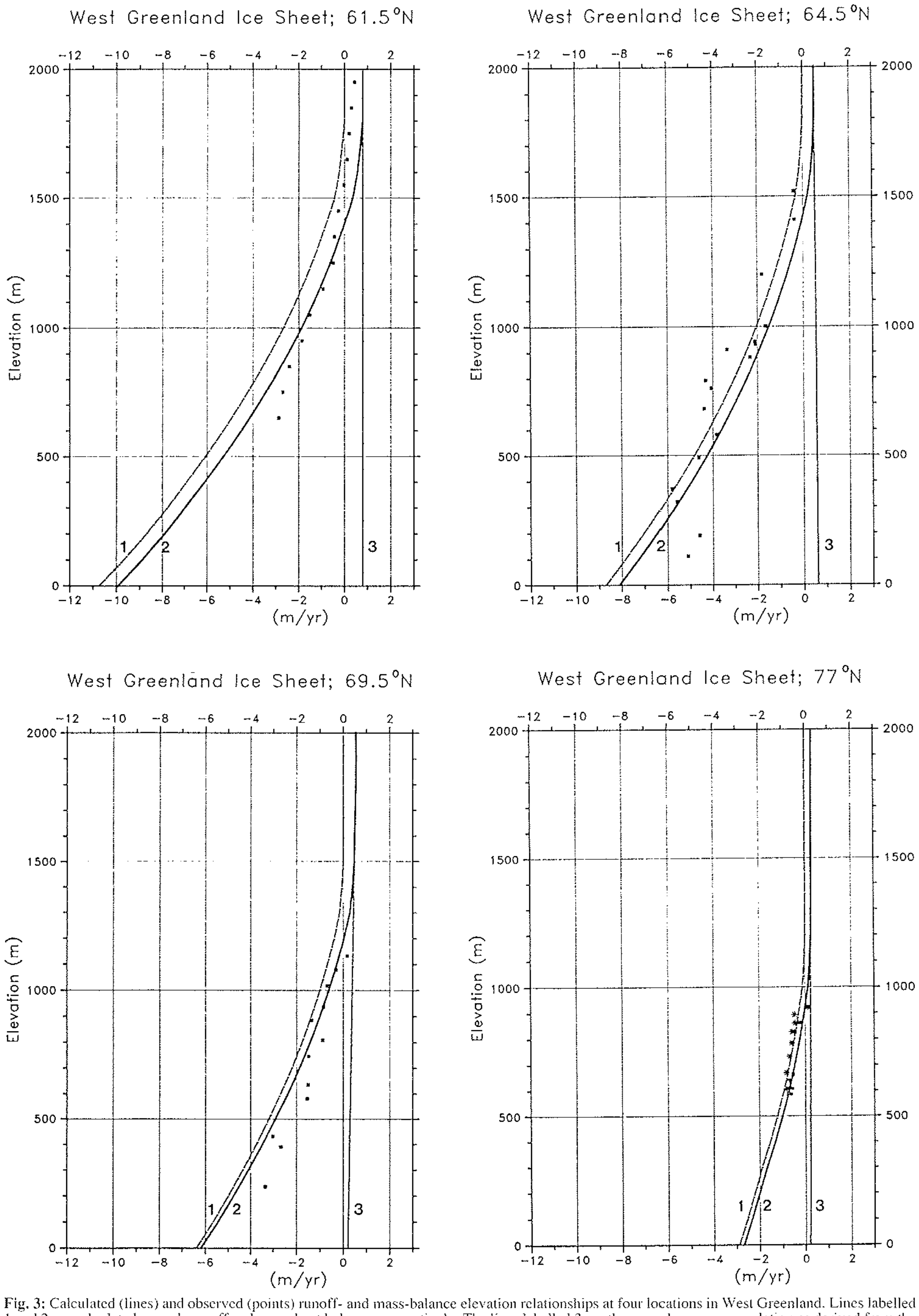

Fig. 3: Calculated (lines) and observed (points) runoff- and mass-balance elevation relationships at four locations in West Greenland. Lines labelled 1 and 2 are calculated annual run-off and annual net balance, respectively. The lines labelled 3 are the annual snow accumulation as derived from the map in Fig. 5. Points are observed annual net balances. Data from Nordbogletscher, 61.5° N (CLEMENT 1983), Qamanârssûp sermia, 64.5° N (BRAITHWAITE 1983), PÂKITSOQ, 69.5° N (THOMSEN 1987), and Nunatarssuaq ice ramp, 77° N (NOBLES 1966).

Abb. 3: Berechnete (Linie) und beobachtete (Punkte) Abhängigkeit von Schmelzwasserabfluß und Massenbilanz von der Höhe, dargestellt für vier Gebiete Westgrönlands. Kurve 1 = berechneter Schmelzwasserabfluß, Kurve 2 = jährliche Nettomassenbilanz. Kurve 3 = aus der Karte in Abb. 5 abgeleitete jährliche Schneeakkumulation. Die Punkte entsprechen der gemessenen jährlichen Nettomassenbilanz. Die Daten stammen vom Nordbogletscher, 61,5° N, (CLEMENT 1983), Qamanârssûp sermia, 64,5° N, (BRAITHWAITE 1983), Pâkitsoq, 69,5° N, (THOMSEN 1987) und Nunatarssuaq ice ramp, 77° N, (NOBLES 1966).

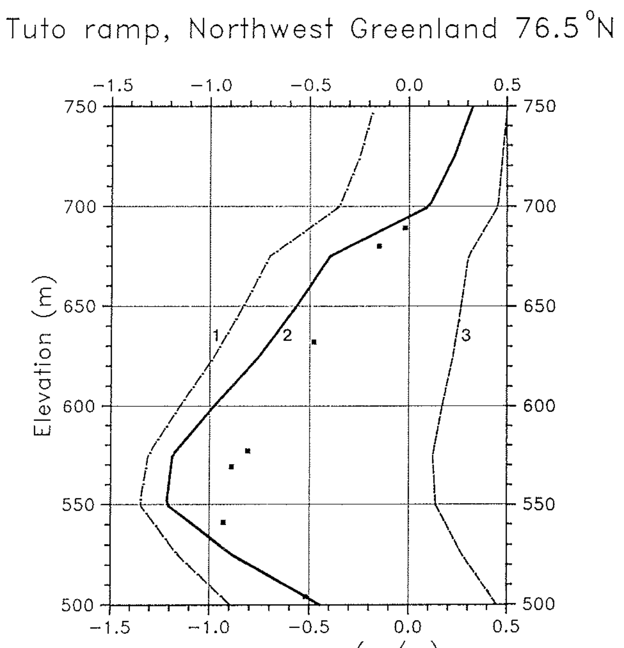

Fig. 4: Calculated (lines 1 and 2) and observed (line 3 and points) runoff- and mass-balance elevation relationships for Tuto ramp. Northwest Greenland. Lines labelled 1 and 2 are calculated annual run-off and annual net balance, respectively. Line labelled 3 is the observed winter balance. Points are observed annual net balances. Data from SCHYTT (1955).

Abb. 4: Für Tuto ramp. Nordwestgrönland, berechnete (Kurven 1 und 2) und beobachtete (Kurve 3 und Punkte) Abhängigkeit von Schmelzwasserabfluß und Massenbilanz von der Höhe. Kurve 1 = berechneter Schmelzwasserabfluß. Kurve 2 = jährliche Nettomassenbilanz. Kurve 3 = gemessene Winterakkumulation. Die Punkte entsprechen der gemessenen jährlichen Nettomassenbilanz. Daten nach SCHYTT (1955).

It appears from Fig. 6 that the elevation range of the superimposed ice zone (SLA-ELA) increases from about 100 m in North Greenland to about 150 m in Southwest Greenland and to about 250 m in Southeast Greenland.

The diagrams shown in Fig. 6 are representative for the conditions on the Greenland ice sheet , and should not be used for local glaciers and ice caps in Greenland, where summer temperatures and snow precipitation may differ greatly from the conditions on the ice sheet. Consequently, the variation of net balance with elevation for the local glaciers may be totally different from the variation indicated in Fig. 6.

6. TOTAL MASS BALANCE OF THE GREENLAND ICE SHEET

Using the present surface elevations of the ice sheet as boundary condition, the total amount of melt from the ice sheet, as determined from the melt-rate model, amounts to 281 km^3/yr, (HUYBRECHTS et al. in press). Integrating the precipitation distribution shown in Fig. 5 over the ice sheet surface yields a total mass input to the ice sheet of 599 km^3/yr. The difference of 318 km^3/yr must leave the ice sheet as calf-ice assuming a steady state. These numbers agree well with the estimates presented by WEIDICK (1984). However, the calculated total melt of 281 km^3/yr is much larger and much more realistic than the value of 69-139 km^3/yr found in the computerized approach of RADOK et al. (1982, p. 118).

Further details about the application of the melt-rate model to estimate the present balance of the Greenland ice sheet and the sensitivity of the balance to climatic change is given by HUYBRECHTS et al. (in press).

7. FIRN WARMING DUE TO RE-FREEZING OF MELTWATER

In the central region of the ice sheet (the dry-snow zone) where melting is insignificant, the surface temperature (TS) is to a good approximation equal to the mean annual air temperature (TMA). However, even for a small amount of melt, the refreezing of percolating melt water in the near-surface snow and firn layers, will cause a rise of the surface temperature in respect to the mean annual air temperature. The difference between the two

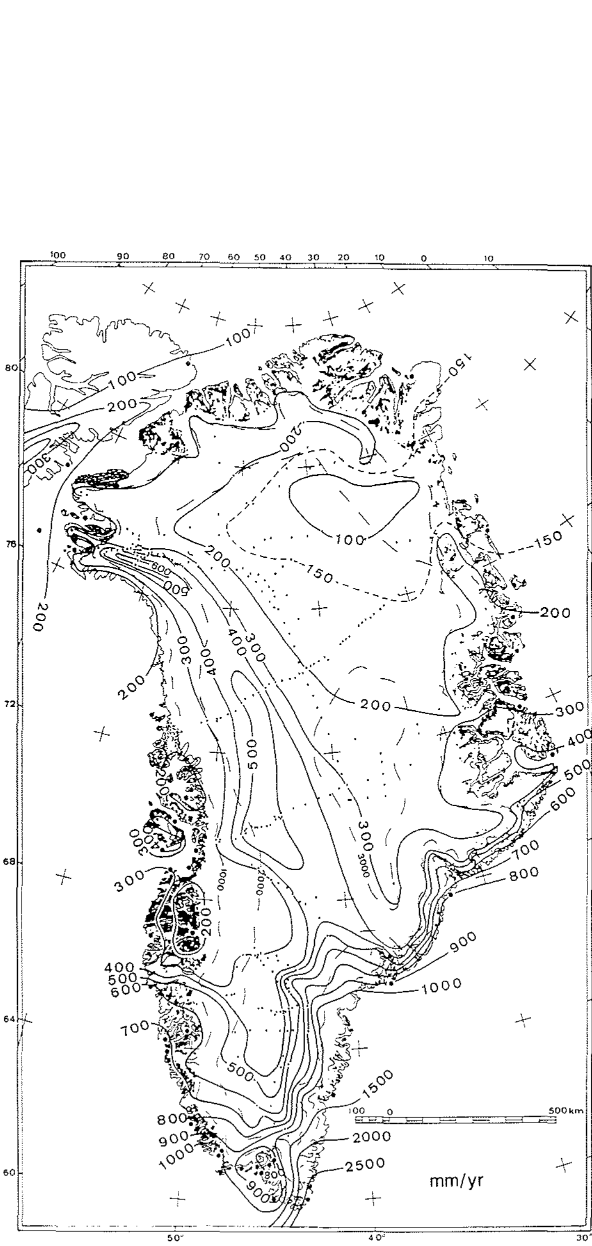

Fig. 5: Distribution of precipitation in Greenland. From OHMURA & REEH (in press).

Abb. 5: Verteilung der Niederschlagsmenge (cm Wasseräquivalent pro Jahr) über Grönland. Nach OHMURA & REEH (im Druck)

temperatures will increase with increasing amount of melt until the point where runoff begins, whereby some of the latent heat escapes from the glacier surface. From this point on, the difference between TS and TMA starts to decrease, and eventually approaches zero somewhere in the ablation zone. In the ablation zone TS may even be lower than TMA because the temperature at the ice surface can not exceed the melting point. Consequently, positive summer temperatures which contribute to TMA, will not contribute to TS. In the ablation zone there is also an „advective" contribution to the surface temperature due to the fact that the re-surfacing ice carries temperature information from upstream regions. A detailed discussion of the relationship between mean annual air temperatures and surface temperatures near and below the equilibrium line can be found in a paper by HOOKE et al. (1983)

The variation with elevation of the mean annual air temperature and the snow/ice surface temperature on the Greenland ice sheet is illustrated in Figs. 7 and 8. In the figures are plotted all observations from West Greenland of mean annual air temperatures and 10-metre snow/ice temperatures in the latitude bands 76-77° N and 69-71° N. In these two latitude bands there is a sufficient number of observations to allow determination of the air-temperature elevation gradient.

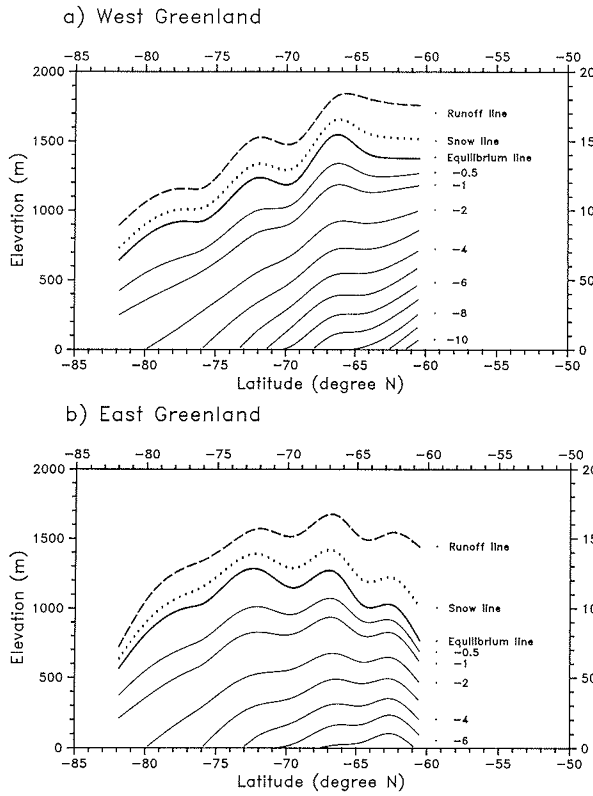

Fig. 6: Calculated net balance for the ablation zone of the Greenland ice-sheet as function of elevation and latitude. The equilibrium line, the snow line, and the runoff line are also shown. (a) West Greenland. (b) East Greenland.

Abb. 6: Berechnete Nettomassenbilanz für das Ablationsgebiet des grönländischen Inlandeises als Funktion von Höhe und geografischer Breite, getrennt für (a) Westgrönland und (b) Ostgrönland. Gleichzeitig dargestellt sind Gleichgewichts-, Altschnee- und Grenzlinie für den Schmelzwasserabfluß.

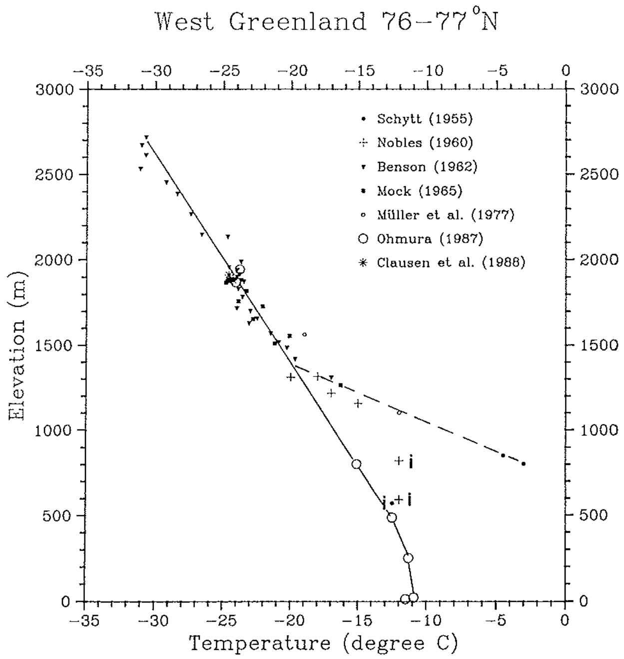

Fig. 7: Observed mean annual air temperatures (large open circles) and snow/ice surface temperatures (other points) on the Northwest Greenland ice sheet. Surface-temperature points marked by (i) are from the ablation zone.

Abb. 7: Beobachtete Jahresmittel der Lufttemperatur (große offene Kreise) und oberflächennahe Firntemperatur (übrige Signaturen) auf dem nordwestlichen grönländischen Inlandeis. Die mit (i) markierten Temperaturwerte stammen aus dem Ablationsgebiet.

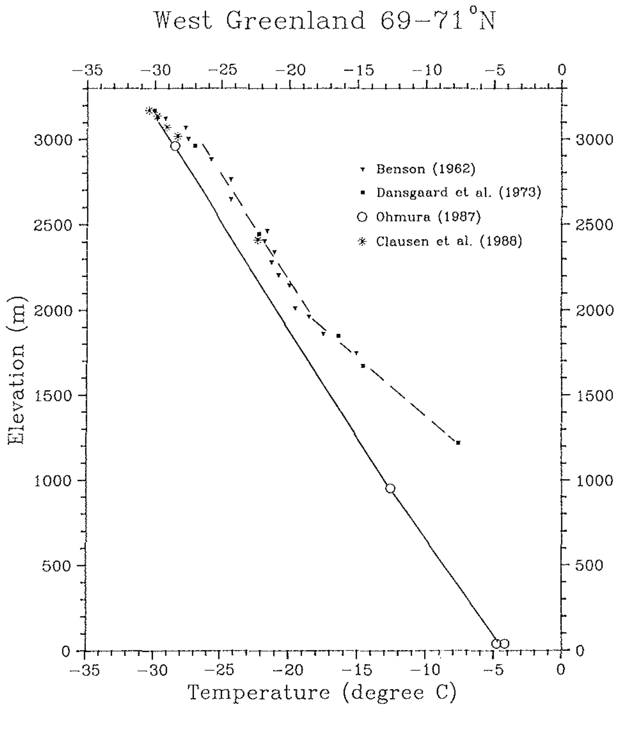

Fig. 8: Observed mean annual air temperatures (large open circles) and snow-surface temperatures (other points) in central West Greenland.

Abb. 8: Beobachtete Jahresmittel der Lufttemperatur (große offene Kreise) und oberflächennahe Firntemperatur (übrige Signaturen) im mittleren Teil Westgrönlands.

It appears from Fig. 7 (76-77° N) that the surface temperature above 1400-1500 m elevation is close to the line defined by the mean annual air temperature. However, below 1400 m the surface temperatures show increasing positive deviations from the air-temperature line, except for the three points marked by (i) which are all from the ablation zone of the ice sheet, and which approach the air-temperature line with decreasing elevation. This is exactly the expected behaviour, c.f the discussion above. Fig. 7 also illustrates the change in air-temperature gradient below about 400 m, caused by the previously mentioned inversion (primarily occurring in the winter) at high latitudes in Greenland.

Fig. 8 (69-71° N) illustrates that there is no indication of such an air-temperature inversion at lower latitudes. The snow-surface temperature above about 1900 m elevation defines a line with the same gradient as the air temperature line, however with an offset of about 2 K. Below 1900 m the gradient changes and the deviation of the surface temperature from the air temperature line increases. Except for the offset of 2 K above 1900 m elevation, this is the expected pattern. As previously mentioned, the air-temperature data have all been referred to the period 1951-1960. All snow-surface temperatures were observed in the late 1950'es except those of CLAUSEN et al. (1988). However, the snow-surface temperature is influenced by the air temperature in the previous 20 year period, which was warmer than the reference period. The observations of CLAUSEN et al. (1988) were performed in 1984-1985, and can therefore be expected to be closer to the air-temperatures in the reference period, as is actually the case for most of these observations. However, this explanation can hardly account for all of the 2 K difference. A contributing cause is probably that BENSON (1962) often measured temperature only to 4 m depth, and calculated the 10-metre temperature by a correction procedure, which seems to have caused an overestimation of the 10-metre temperature. It is not obvious, why a similar offset is not found in the data from 76-77° N. Nevertheless, both Fig. 7 and Fig. 8 illustrate the increasing firn warming with decreasing altitude, in Northwest Greenland below about 1400 m and in central West Greenland below about 1900 m.

Fig. 9 displays the firn-warming DT = TS - TMA at all Greenland ice sheet stations versus the amount of refrozen meltwater, i.e. the amount of superimposed ice formed (SIF). DT is determined as the difference between the observed 10-metre snow temperature and the mean-annual air temperature as calculated from Equation (1). SIF

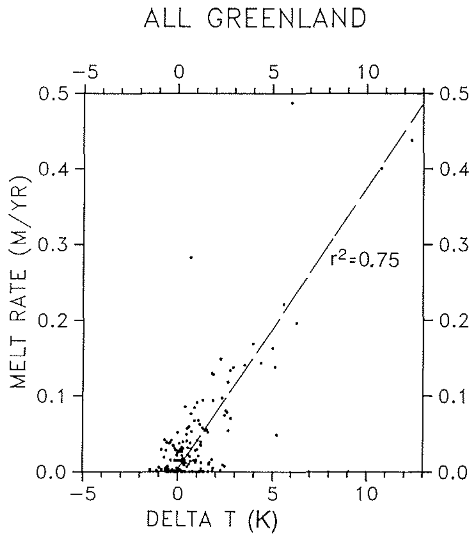

Fig. 9: Relationship between firn warming (positive deviation of the observed snow/ice surface temperature from mean annual air temperature as calculated from Equation (1)) and annual melt rate (formation of superimposed ice, SIF) on the Greenland ice sheet. SIF is calculated using the snow-and-ice-melt model. Surface-temperature data from MOCK &WEEKS (1965), SCHYTT (1955), NOBLES (1960), DANSGAARD et al. (1973), MÜLLER et al.(1977), and CLAUSEN et al. (1987).

Abb. 9: Beziehung zwischen Firnerwärmung (positive Abweichung der beobachteten oberflächennahen Firn-/Eistemperatur vom Jahresmittel der Lufttemperatur, berechnet nach Gleichung (1)) und jährlicher Schmelzrate (Bildung von Aufeis, SIF) für das grönländische Inlandeis. SIF wurde mit dem Schnee- und Eisschmelzmodell berechnet. Daten für die Firn- und Eistemperaturen nach MOCK & WEEKS (1965), SCHYTT (1955), NOBLES (1960), DANSGAARD et al. (1973), MÜLLER et al. (1977) und CLAUSEN et al. (1987).

is calculated by means of the snow-and-ice-melt model described in a previous section. In spite of a large scatter, the diagramme shows the expected trend of increasing DT with increasing SIF. Disregarding one point that deviates from the general pattern, a linear regression ($r^2 = 0.75$) suggests the following relationship between DT (K) and SIF (m/yr)

$$DT = 0.86 + 26.6\ (SIF - 0.038)$$

which within the accuracy limits may also be written

$$DT = 26.6\ SIF \qquad (5)$$

In the percolation zone the surface temperature can accordingly be calculated as

$$TS = TMA + 26.6\ SIF$$

where TMA is determined by Equation (1) and SIF is calculated by means of the snow-and-ice-melt model described in a previous section. In Fig. 10 the firn warming DT = TS - TMA at all Greenland ice sheet stations is plotted versus elevation (points marked by sqares). Plots are made for 5 different latitude bands, each spanning two degrees of latitude. TS is the observed 10-metre surface temperature, whereas TMA is estimated, using the parameterization given by Equation (1). The full curves also drawn in Fig. 10 show the firn warming as a function of elevation as estimated from Equation (5), with SIF calculated from the snow-and-ice-melt model. As is apparent from Fig. 10, there is some scatter of the points around the curves. Part of the scatter can be explained by the fact that the surface temperatures have been measured in different years during the past 30-40 year period. Climatic temperature variations in this period are likely to have caused variations in the 10-metre firn temperature of 1-2 degrees. However, in all latitude bands, the points follow the trends of the curves, suggesting that the snow-and-ice-melt model does a reasonably good job also in determining the relatively modest melt rates in the ice sheet regions above the equilibrium line.

It appears from Fig. 10 that a significant firn warming (more than 1 degree) and consequently a significant summer melting occurs even at the highest elevations (about 2800 m) of the South Greenland ice sheet. In North Greenland a firn warming of less than one degree is found above an elevation of about 1650 m. Consequently, negligible firn warming (summer melting) occurs over a large area of the North Greenland ice sheet above this elevation. These results agree with the distribution of the dry-snow facies as given by BENSON (1962). A more detailed comparison of the present results with Bensons distribution of diagenetic facies on the Greenland ice sheet is in preparation.

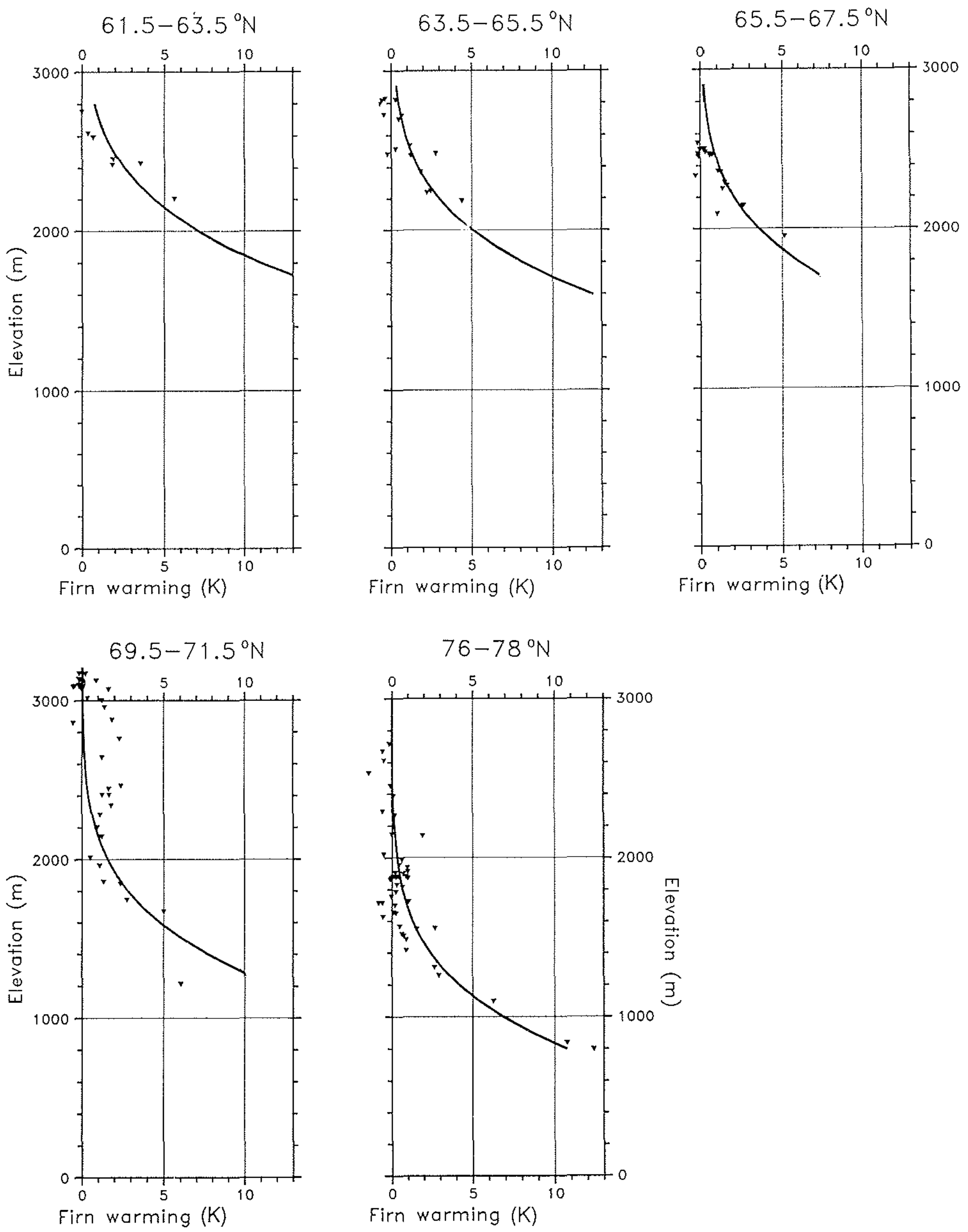

Fig.10: Firn warming on the Greenland ice sheet as a function of elevation in different latitude bands. Points are determined as the deviation of the observed snow-surface temperature from the mean annual air temperature as calculated from Equation (1). The curves are calculated by means of Equation (5) with SIF determined by using the snow-and-ice-melt model. Surface-temperature data from MOCK & WEEKS (1965), SCHYTT (1955), NOBLES (1960), DANSGAARD et al. (1973), MÜLLER et al. (1977), and CLAUSEN et al. (1987).

Abb. 10: Firnerwärmung als Funktion der Höhe, dargestellt für verschiedene Breitenbereiche des grönländischen Inlandeises. Die Punkte repräsentieren die positive Abweichung der beobachteten oberflächennahen Firn-/Eistemperatur vom Jahresmittel der Lufttemperatur, berechnet nach Gleichung (1). Die Kurven wurden nach Gleichung (5) berechnet, wobei SIF mit dem Schnee- und Eisschmelzmodell bestimmt wurde. Die Daten für die Firn- und Eistemperaturen stammen von MOCK & WEEKS (1965), SCHYTT (1955), NOBLES (1960), DANSGAARD et al. (1973), MÜLLER et al. (1977) und CLAUSEN et al. (1987).

CONCLUSIONS

It has been shown that parameterizations of mean-annual and mean-July air temperatures combined with a snow-and-ice-melt model which relates melt to positive degree-days provide a fairly accurate description of the present melt rates and 10-metre surface temperatures on the Greenland ice sheet, as far as these quantities are known today. Besides providing the necessary surface boundary conditions for ice sheet dynamic model studies (LETRÉGUILLY et al. in press), the model has been used to estimate the total surface mass balance of the Greenland ice sheet. Since the model involves very few parameters it is also easy to investigate how surface mass balance will change in a changing climate. The model can, therefore, be used to estimate the sensitivity of the surface mass balance of the Greenland ice sheet to climatic change, e.g. the expected greenhouse warming (HUYBRECHTS et al. in press).

The model can be improved on several points: More air-temperature data from the Greenland ice sheet, particularly from East and North Greenland where such data are scarce, would help to improve the temperature parameterizations. Studies of the melting-refreezing process (the formation of superimposed ice) near the equilibrium line would help to improve the snow-and-ice-melt model. And finally, more net-balance studies in the ablation zone of East and North Greenland from where very few data are available would provide new information against which to check the model.

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