# Weighting of satellite Mean Dynamic Topography data in inverse ocean models

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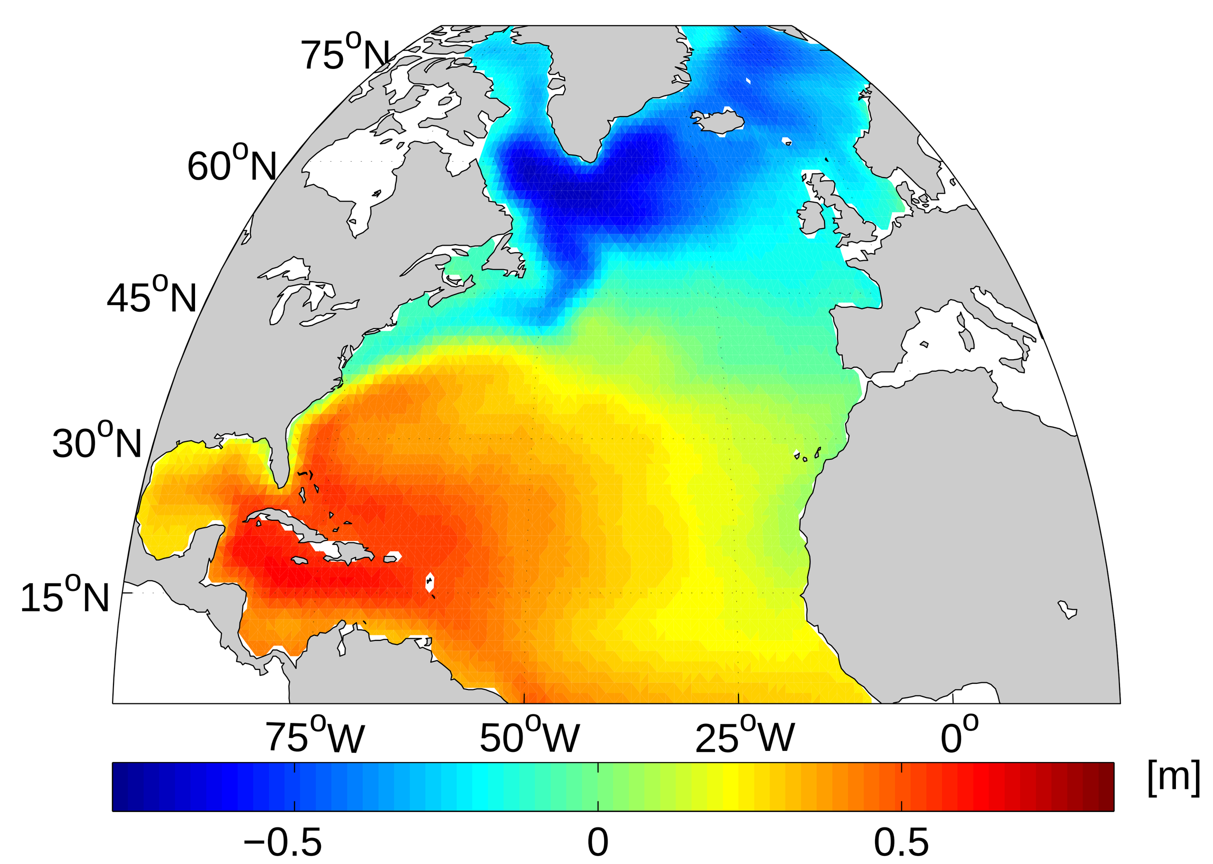
## Introduction

Ocean models can be improved by the assimilation of Mean Dynamic Topography (MDT) data. (In the geostrophic approximation, the MDT is equivalent to ocean surface velocity.) The inverse ocean model IFEOM assimilates MDT data $\eta_{\text{data}}$ from satellite observations.

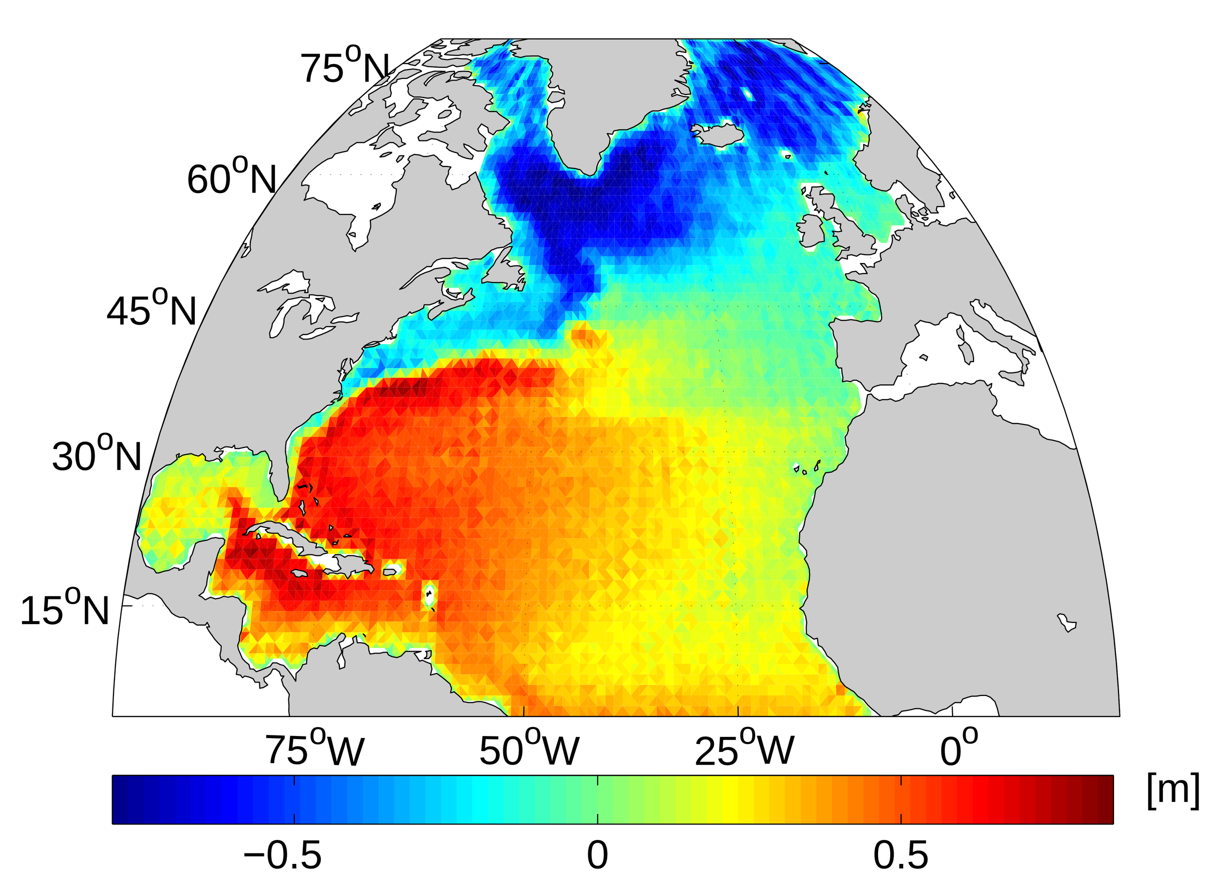
Minimization of cost function: $J = \frac{1}{2}\sum_i J_i$,
with $i$=temperature T, salinity S, velocities v, MDT $\eta$,... and

$$J_\eta = (\eta_{\text{model}} - \eta_{\text{data}})^T \mathbb{W}_\eta (\eta_{\text{model}} - \eta_{\text{data}}).$$

$\mathbb{W}_\eta = \mathbb{C}_\eta^{-1}$ is the inverse MDT error covariance. In our case, $\mathbb{W}_\eta$ is a dense matrix and is provided along with the data.




First guess MDT $\eta_{\text{model}}$ (IFEOM without assimilation of $\eta_{\text{data}}$)




Satellite MDT $\eta_{\text{data}}$

## Problem: Weighting the cost function terms

In theory, $\mathbb{C}_\eta^{-1} = \mathbb{W}_\eta$ should be used as the weighting matrix for the MDT data $\eta_{\text{data}}$ in the optimization.

In reality, the MDT data $\eta_{\text{data}}$ is heavily overweighted by this $\mathbb{W}_\eta$ ☹

Possible reasons:
- (unknown) model errors
- poor error (covariance) estimate
- ...

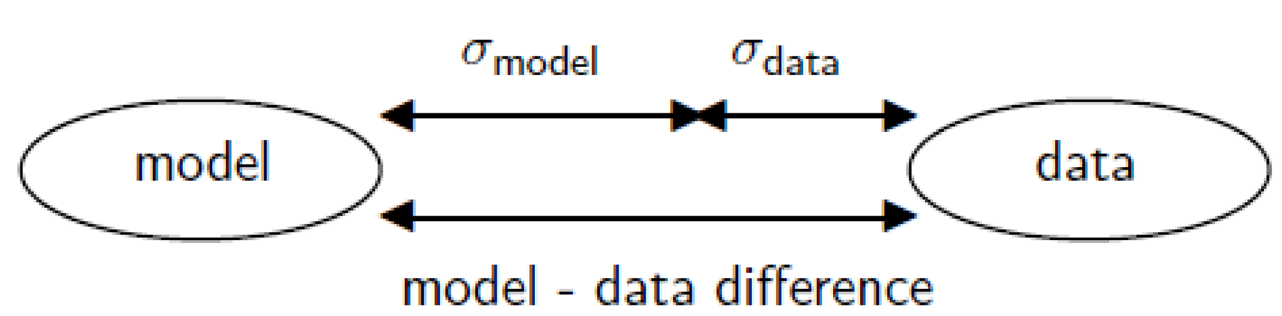
## Workaround: Determine weighting factor $\alpha$:

$$J_\eta = \frac{1}{\alpha} \cdot (\eta_{\text{model}} - \eta_{\text{data}})^T \mathbb{W}_\eta (\eta_{\text{model}} - \eta_{\text{data}}).$$

A weighting factor $\alpha$ is introduced to reduce the weight on the MDT data. Three approaches are tested for a justifiable downweighting:

## Approach 1: Minimum model MDT error

Reasonable model–data differences should be smaller than the sum of model standard deviation $\sigma_{\text{model}}$ and data standard deviation $\sigma_{\text{data}}$:

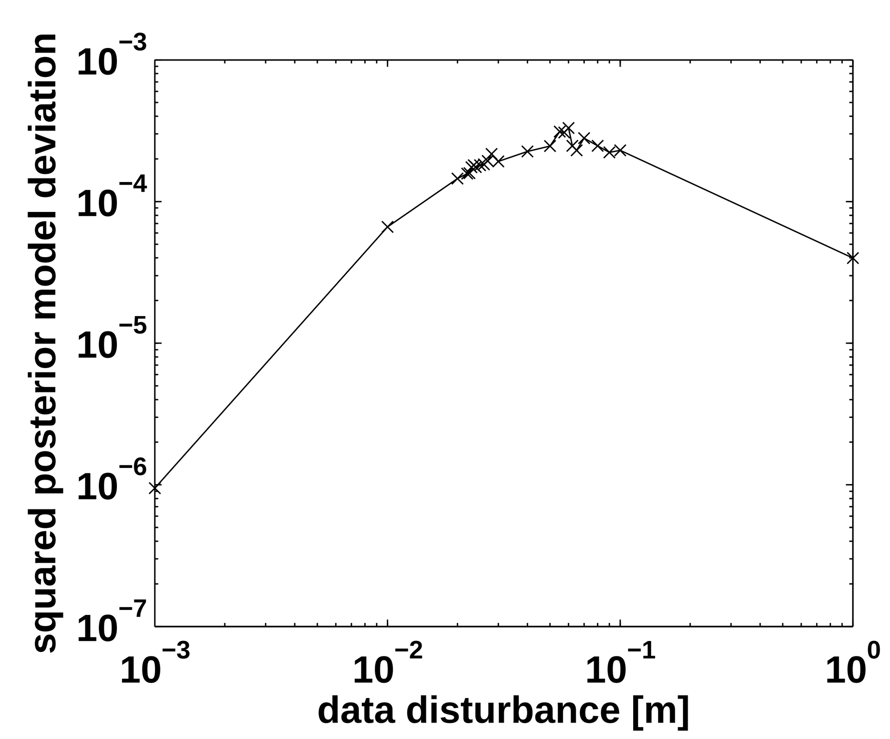



$$\Rightarrow \quad \alpha > 3.3$$

This approach provides only a lower boundary.

## Approach 2: Maximum model entropy

Find data error that maximizes model entropy:

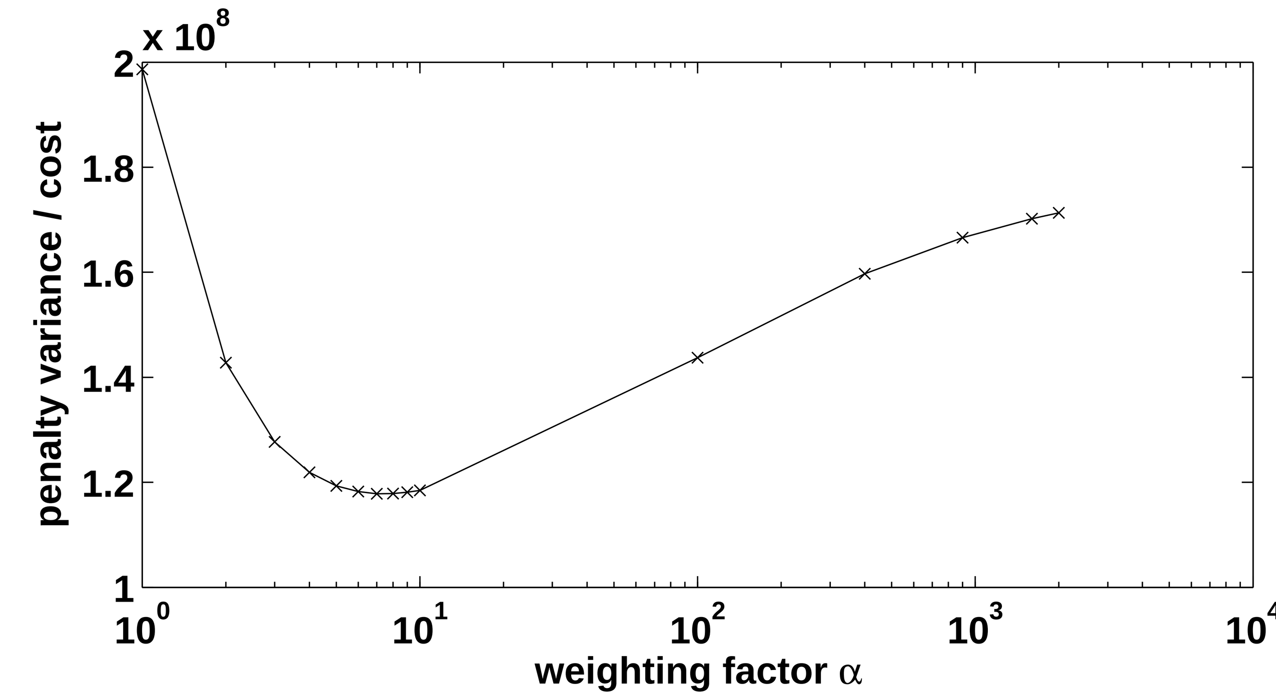



$$\Rightarrow \quad \alpha = 30$$

This $\alpha$ is too large: The optimized MDT is almost identical to the first guess MDT.

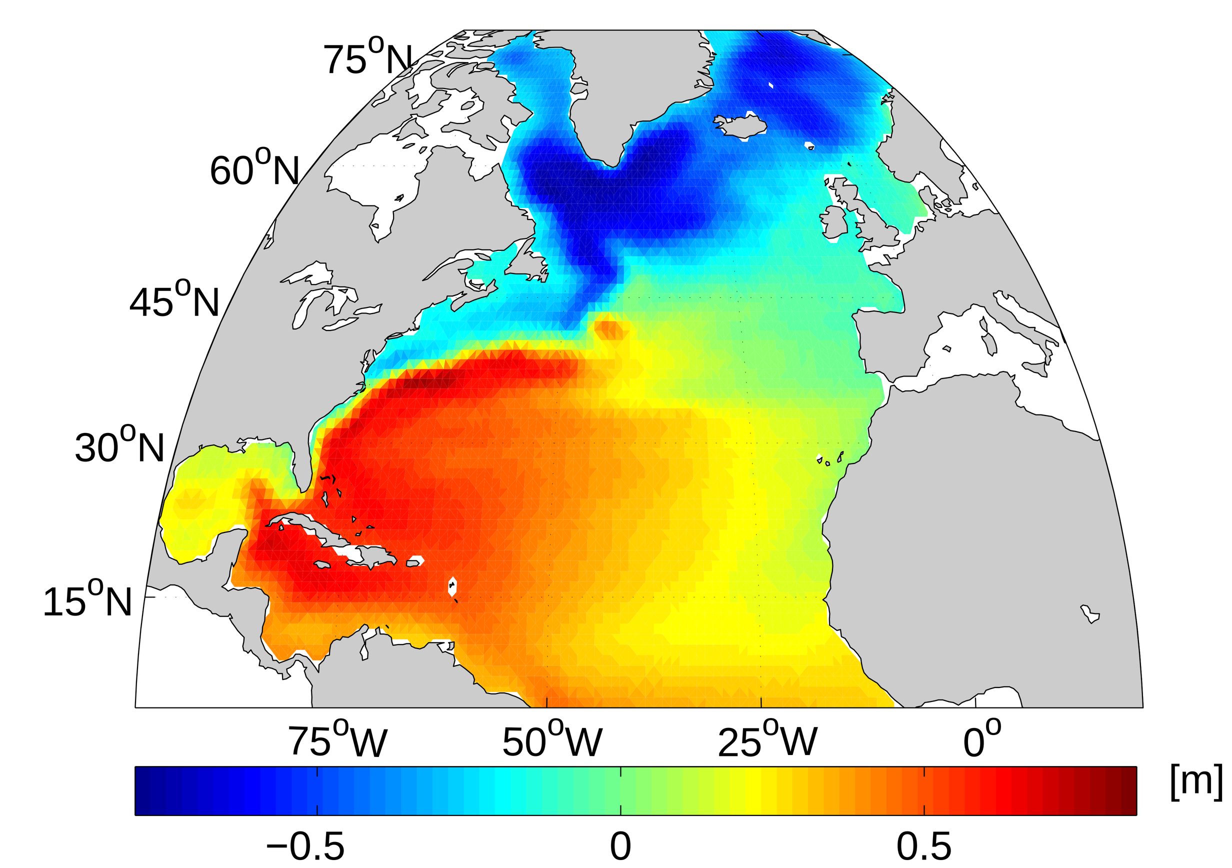
## Approach 3: Minimum penalty variance

Penalty variance of the cost function terms (for $T$, $S$, $\eta$, ...) (normalized by overall cost) in dependence of weighting factor $\alpha$:




$$\Rightarrow \quad \alpha = 7$$

## Result: Optimized MDT with $\alpha = 7$




The optimized MDT (figure above) using the weighting factor $\alpha = 7$ from the minimum penalty variance approach is a reasonable trade-off between the first guess of the model $\eta_{\text{model}}$ and the data $\eta_{\text{data}}$.
The unphysical noise from the data $\eta_{\text{data}}$ has disappeared. The Gulf Stream is intensified compared to the first guess, and the Mann Eddy is present in the solution.

## Summary


- Existing theory is not sufficient for weighting of the MDT data–model combination.
- Different approaches for a justifiable weighting method are theoretically possible.
- For this specific model–data combination, the minimum penalty variance approach leads to a reasonable weighting factor $\alpha$.
- The result of the optimization is improved by the new method.