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Modelling the Transformation from Snow to Ice Based on the Underlying Sintering Process

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In cold regions of the earth, like Antarctica, Greenland or mountains at high altitude, the annual amount of deposited snow exceeds the amount of snow melting. Snow, which is more than one year old, is called firn. Over time firn transforms into ice by a sintering process, mainly driven by overburden pressure and temperature. This ultimately leads to the formation of glaciers and ice sheets.

We simulate firn densification based on the processes of sintering. The constitutive law represents grain boundary sliding, dislocation creep and diffusion. These mechanisms sum up to the overall densification which leads to the transformation of snow to ice. The model aims at obtaining a physics driven simulation tool for firn densification which provides data for a wider range of areas. It will contribute to develop better models and better understanding of the cryosphere.

1 Introduction & Firn Densification

Firn occurs in the polar regions and at high altitudes, where snow survives more than one year without being ablated. Typically it exhibits a density of about $\rho_0 \approx 320 - 360\,\mathrm{kg\,m^{-3}}$ at the surface [1], increasing with depth until ice density $\rho_{\mathrm{ice}} = 917\,\mathrm{kg\,m^{-3}}$ is reached. In natural conditions on earth ice has always a temperature near its pressure melting point, even in ice sheets. One phenomenon that demonstrates this is the process of firn densification, which is similar to so called hot isostatic pressing (HIPing) [2], a process described in the context of sintering ceramics. HIPing can be divided in two stages [3]. An additional stage preceding the ones of HIPing is added for the description of firn densification at low densities.

The first stage of firn densification takes place until a relative density of about $\varrho = \frac{\rho}{\rho_{\mathrm{ice}}} = 0.6$ or an absolute density of approximately $\rho = 550\,\mathrm{kg\,m^{-3}}$ is reached. In this stage single grains of ice are just loosely connected. Sporadic aggregates can form. At the end of this stage a theoretical sphere packing is reached. The second stage of firn densification and HIPing begins. The second densification stage ends at a relative density of $\varrho_{\mathrm{c}} = 0.9$ or about $\rho_{\mathrm{c}} = 834\,\mathrm{kg\,m^{-3}}$ when the pores between grains are closed off (see Fig. 1, [4]). The third stage begins.

In these three stages the densification is controlled by different processes to different degrees. Grain boundary sliding (GBS) describes how different grains of ice in the loosely packed firn of the first densification stage slide against each other [5]. Wilkinson & Ashby (1975) [6] applied power law creep to firn for the description of dislocation creep (DC). Dislocation creep is driving most of the densification in stages two and three, but we suggest to consider it also in stage one. In the last stages two diffusion processes contribute significantly: Boundary diffusion (BD) and Lattice diffusion (LD) describing Coble creep and Nabarro-Herring creep like mass transport between the ice grains [7].

2 Model

The model approach follows the lines of standard continuum mechanics and is solving the balance equations, conservation of mass, linear momentum and energy in vertical direction. The constitutive law follows the concept that the overall strain rate is the sum of the strain rates obtained by individual constitutive equations for the four processes of firn densification:

$$\dot{\varepsilon}_{zz} = \dot{\varepsilon}_{\mathrm{GBS}} + \dot{\varepsilon}_{\mathrm{DC}} + \dot{\varepsilon}_{\mathrm{BD}} + \dot{\varepsilon}_{\mathrm{LD}} \quad . \tag{1}$$

We do not discuss boundary and lattice diffusion here in detail, as their influence on the densification rate is much smaller than that of dislocation creep. They can be found in Coble (1970) [7], Maeno & Ebinuma (1983) [2] and Arthern & Wingham (1998) [8]. Equation (2) shows the constitutive law for grain boundary sliding [5]:

$$\dot{\varepsilon}^{\mathrm{I}}_{\mathrm{GBS}} = -\frac{16}{15}\frac{D_{\mathrm{DBC}}\Omega\delta_b}{k_B T h^2}\frac{1}{r\mu^2}\left(\frac{\rho_{\mathrm{ice}}}{\rho}\right)^3\left(1-\frac{5}{3}\frac{\rho}{\rho_{\mathrm{ice}}}\right)(t_{zz} - t_{0\,zz}) \quad . \tag{2}$$

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Where D_{DBC} describes a temperature dependent Arrhenius law, $\Omega = 3.27 \times 10^{-29}\,\mathrm{m}^3$ is the volume per H_2O molecule, $\delta_b = 9 \times 10^{-10}\,\mathrm{m}$ the width of the boundary of a grain, k_B Boltzman's constant, T is the temperature, $h = 4 \times 10^{-6}\,\mathrm{m}$ the amplitude of grain boundary obstructions, r the grain radius, and $\mu = 0.7$ is the ratio of neck radius to grain radius [5,8]. Grain boundary sliding, as well as dislocation creep, is driven by the stress in vertical direction t_{zz} originating from the overburden firn. As the surface is expected to be traction free, surface stress $t_{0\,zz}$, representing air pressure, is subtracted. The constitutive law for dislocation creep [6] is given by

$$\dot{\varepsilon}_{\mathrm{DC}}^{\mathrm{I+II+III}} = -f_{\mathrm{DC}} A_{\mathrm{DC}} \frac{\left(1 - \frac{\rho}{\rho_{\mathrm{ice}}}\right)}{\left(1 - \left(1 - \frac{\rho}{\rho_{\mathrm{ice}}}\right)^{\frac{1}{n}}\right)^n} \left(f_{\mathrm{DC}} \frac{1}{n} \left(t_{zz} - t_{0\,zz}\right)\right)^n \tag{3}$$

where f_{DC} is a geometric factor which is $f_{\mathrm{DC}}^{\mathrm{I+II}} = 2$ for stages one and two and $f_{\mathrm{DC}}^{\mathrm{III}} = \frac{3}{2}$ for the third stage. When reaching stage three of the densification, at which pores are closed off, the stress at the surface representing air pressure $t_{0\,zz}$ is multiplied with the Boyle-Mariotte equation $\frac{\rho(\rho_{\mathrm{ice}} - \rho_{\mathrm{c}})}{\rho_{\mathrm{c}}(\rho_{\mathrm{ice}} - \rho)}$ representing the increase in pore pressure with increasing density [8,9]. The flow exponent for ice is $n = 3$.

3 Results & Discussion

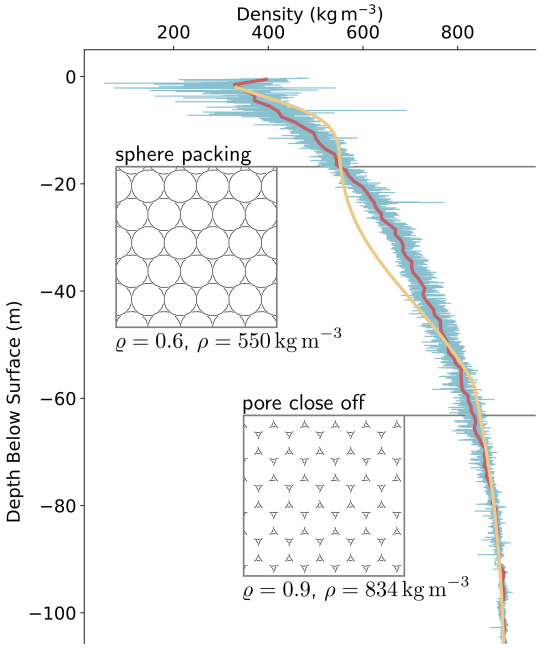

Fig. 1 Depth over density for firn core ngt14C92.2 (Greenland) [11]. The red curve shows mean densities for a window of one meter depth. The yellow line represents the simulated firn profile (temperature $T = 241.77\,\mathrm{K}$, surface accumulation $a_0 = 123.56\,\mathrm{mm\,weq.}^{-1}$, surface density $\rho_0 = 300\,\mathrm{kg\,m^{-3}}$). At a density of $\rho = 550\,\mathrm{kg\,m^{-3}}$ ice grains form a theoretical sphere packing. Pores close off at a density of $\rho_{\mathrm{c}} = 834\,\mathrm{kg\,m^{-3}}$.

Figure 1 displays the density of ice core "ngt14C92.2" obtained in central Greenland [11]. The blue line represents the measured data. The variability, even down to great depth, can be explained by impurities enclosed in the firn [12]. The mean density, averaged over the depth of one meter, is shown in red. At a depth of about $17\,\mathrm{m}$ the relevant density of $\rho = 550\,\mathrm{kg\,m^{-3}}$ is reached. The core shows pore closure density at a depth of approximately $63\,\mathrm{m}$ below surface. The yellow line shows a simulated density curve. Input parameters for the simulation were chosen to fit the conditions at the location of ice core "ngt14C92.2".

The calculated density shows good agreement with the measured data regarding the transition depths between the three stages of firn densification. However, the densification rate is overestimated in the first stage while it is underestimated in the second stage, especially at its beginning. Stage three shows again a good agreement.

The differences may originate from the simplified one dimensional description of the load situation, missing processes occurring during the densification or an inappropriate choice of parameters. The latter is especially true for the process of grain boundary sliding. Future work will investigate this issue.

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