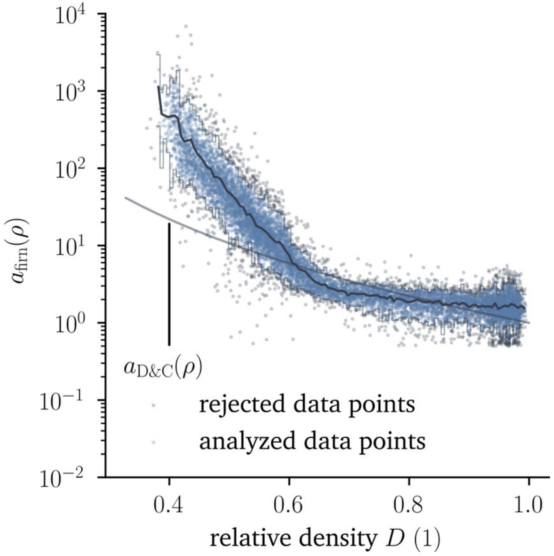
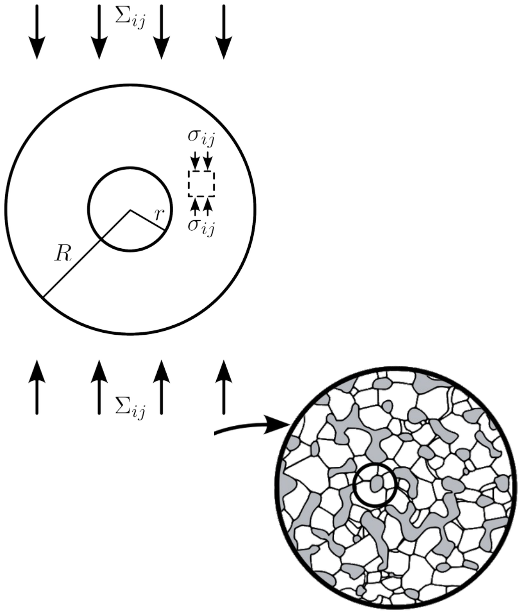
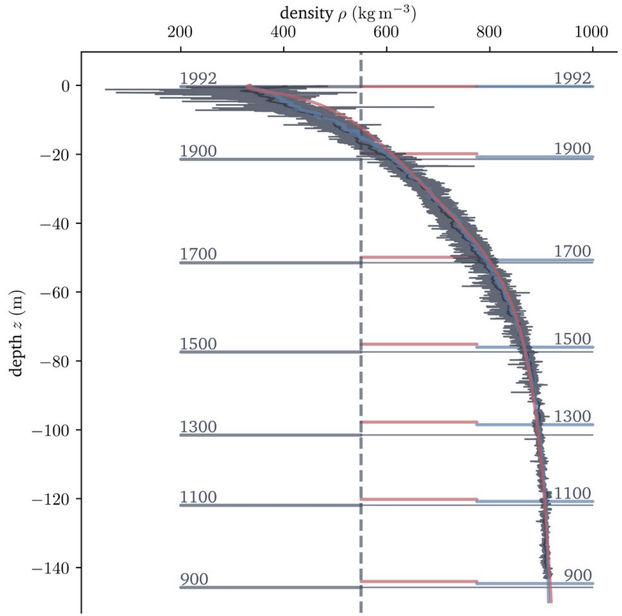
A continuum mechanics perspective on the rheology of firn in the context of firn densification

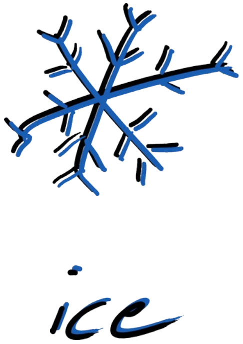




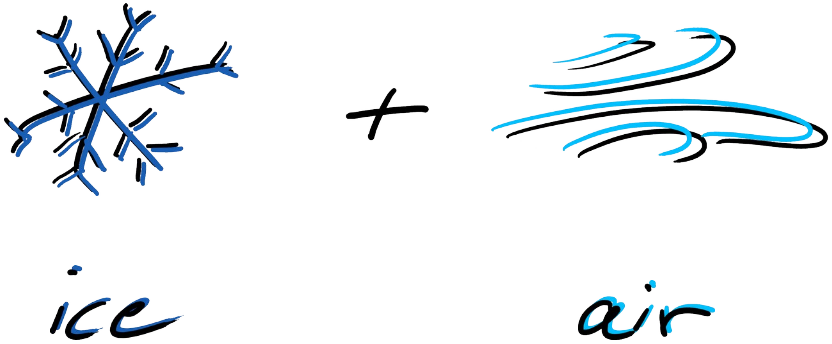


What is firn?

What is firn?



What is firn?



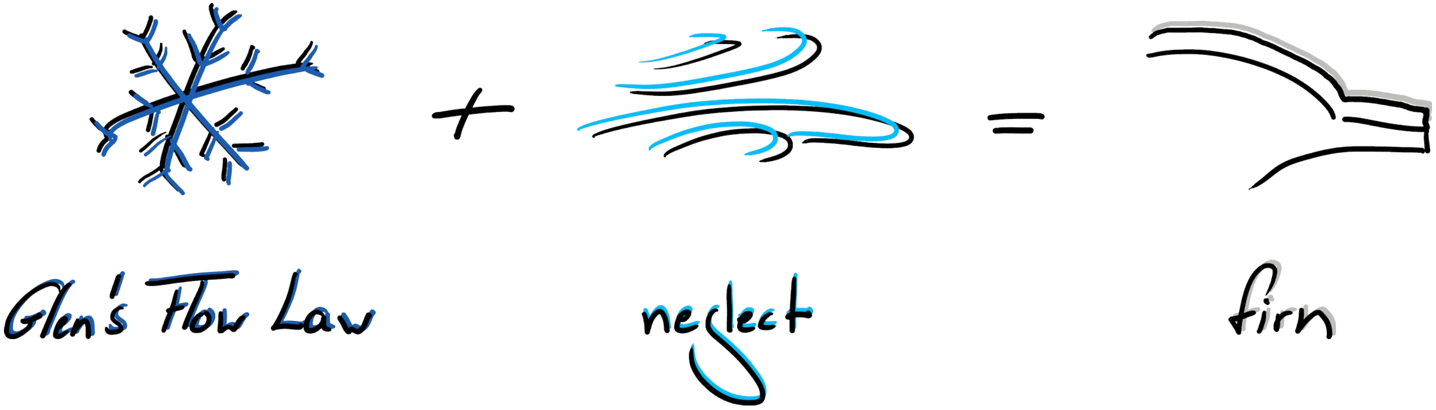
What is firn?

How can we describe the rheology of firn?

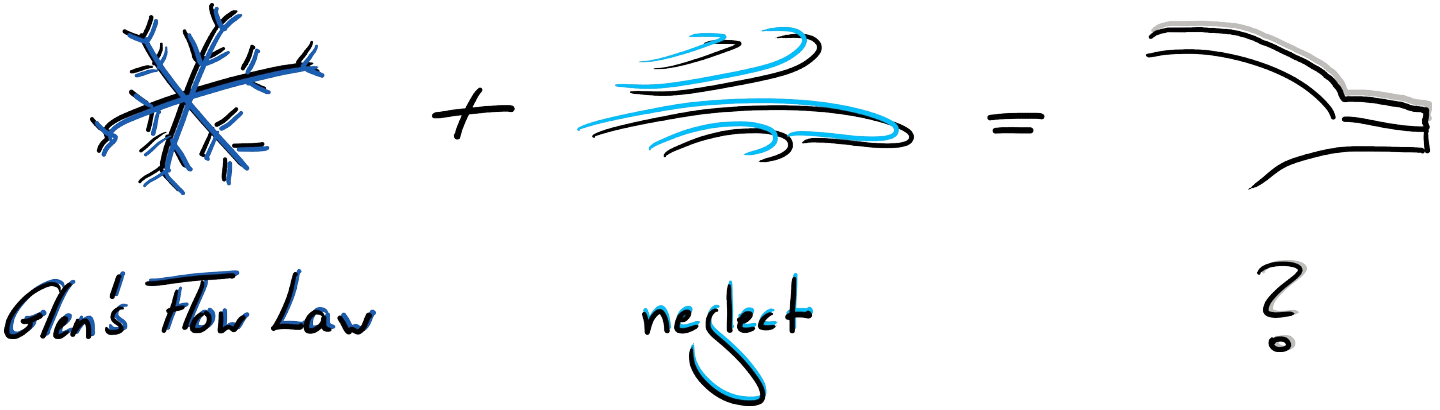
How can we describe the rheology of firn?

A continuum mechanics perspective on the rheology of firn

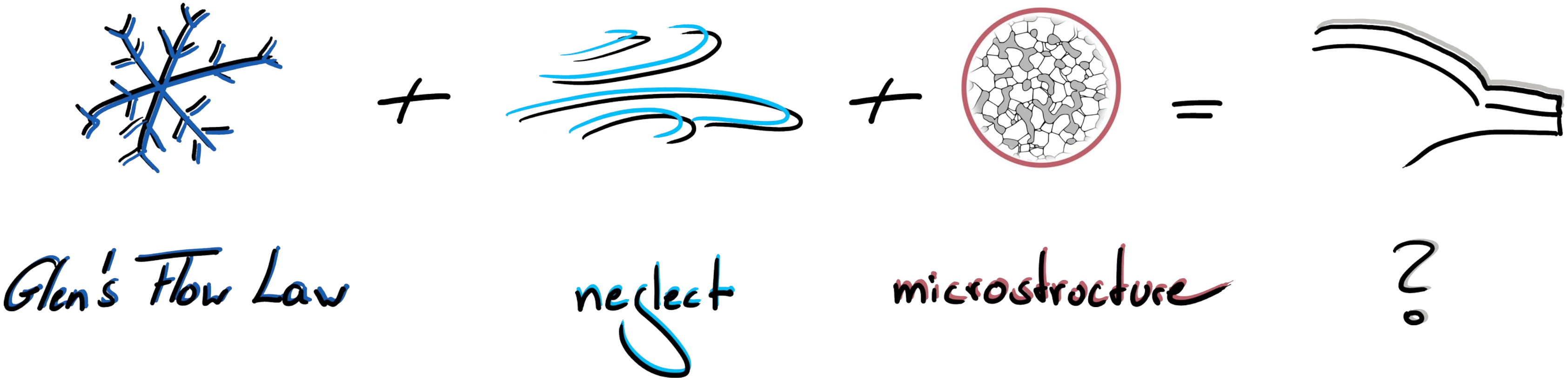
How can we describe the rheology of firn?



How can we describe the rheology of firn?

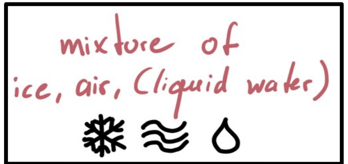


How can we describe the rheology of firn?

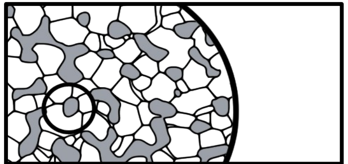


Timm Schultz, Angelika Humbert, and Ralf Müller

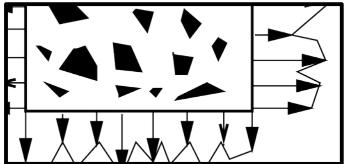
Contents



The Material



Micro-Macro



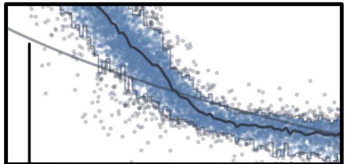
Cell-Model

$$S = (a(\rho)\,\Sigma_e^2 +$$

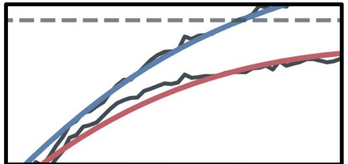
Effective Stress

$$\zeta\left(\rho, T, \dot{E}_{ij}\right)$$

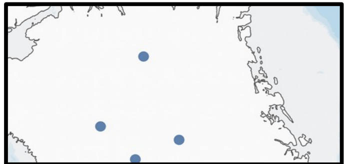
Material Model



Model Inversion



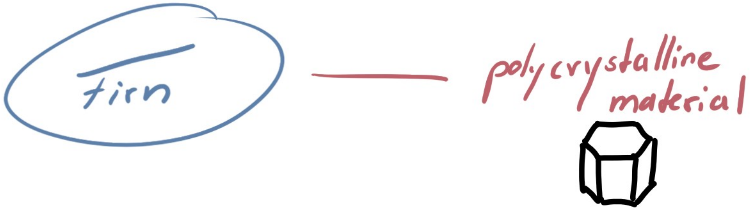
Material Behavior



Examples

The Material Firn

The Material Firn



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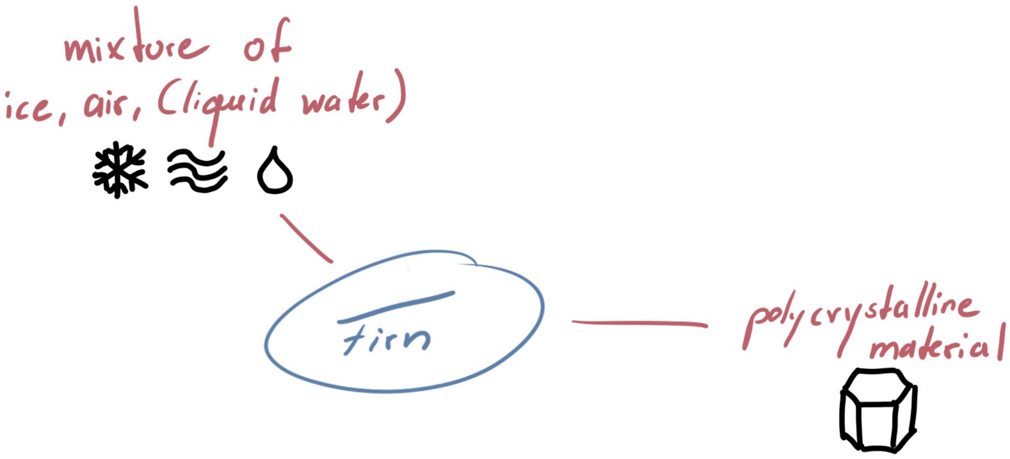

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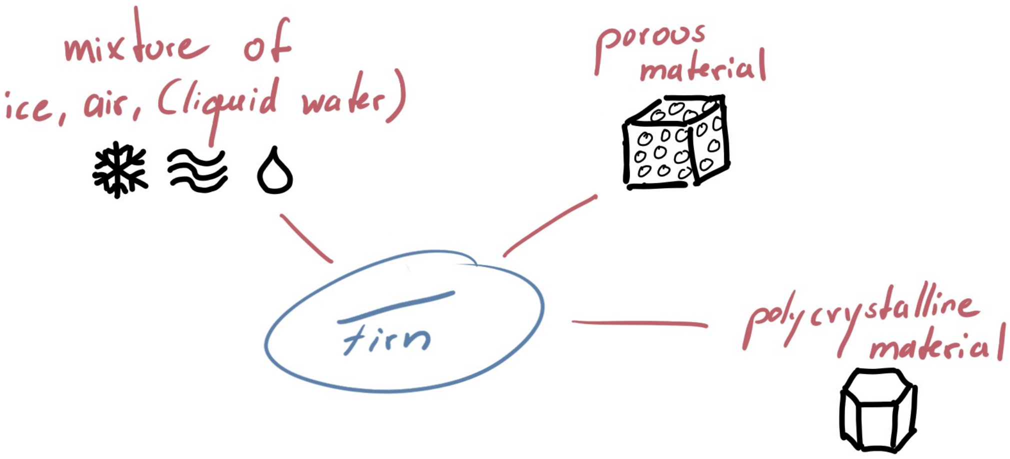

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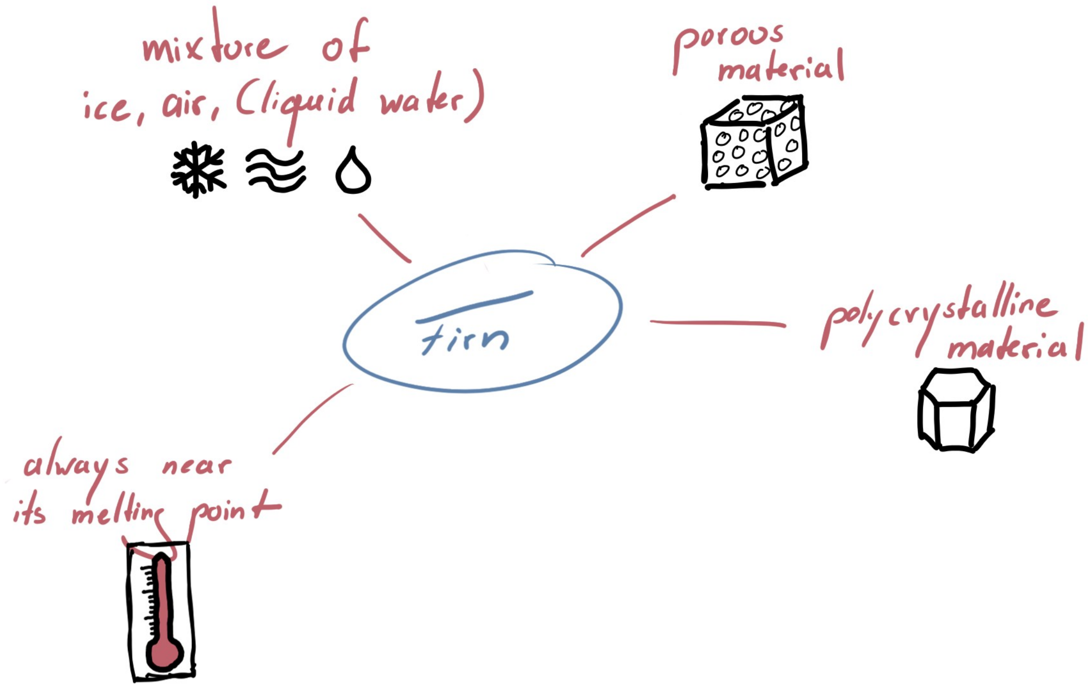

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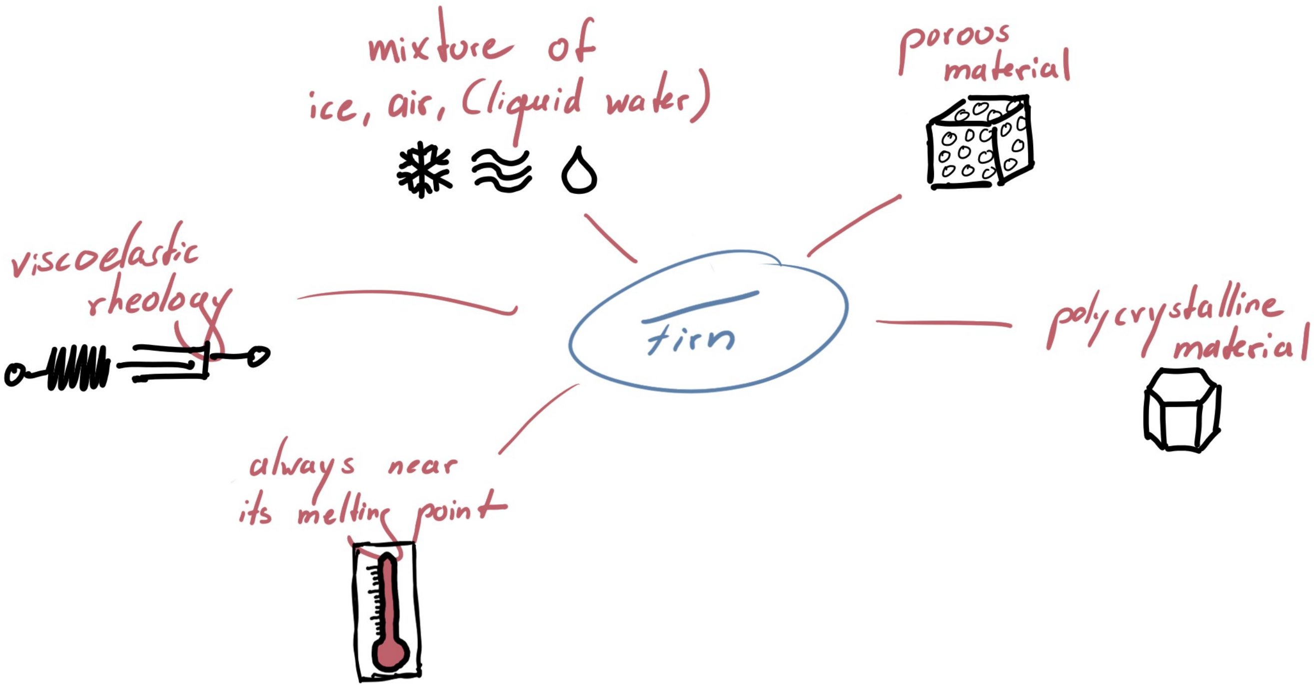

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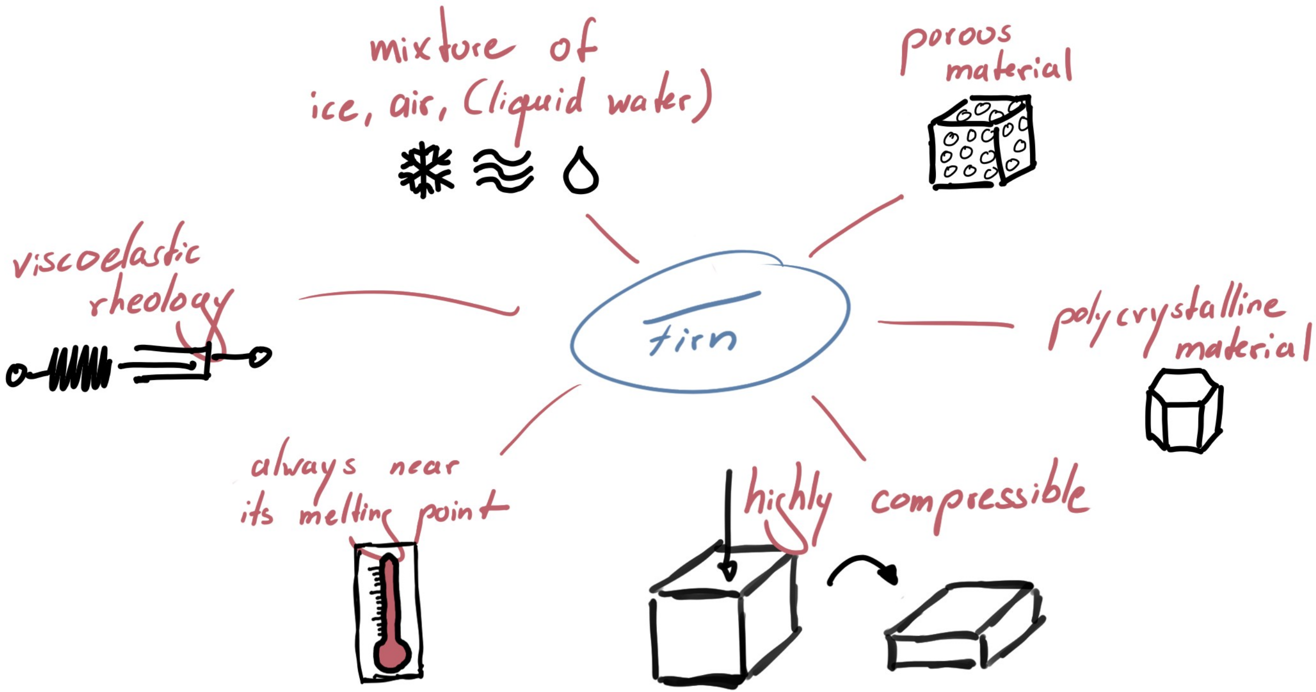
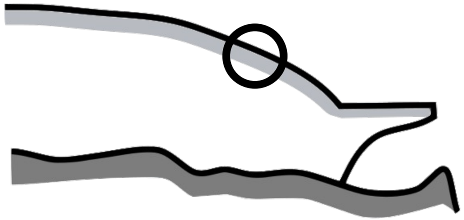


From the macro-scale to the micro-scale



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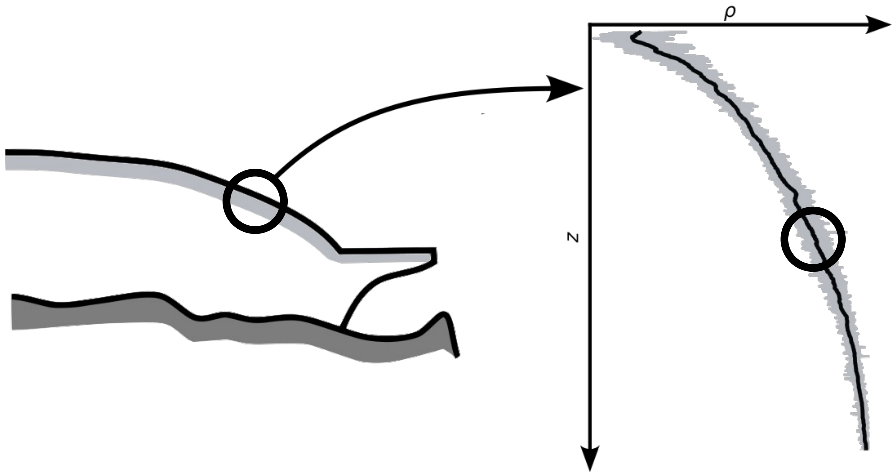

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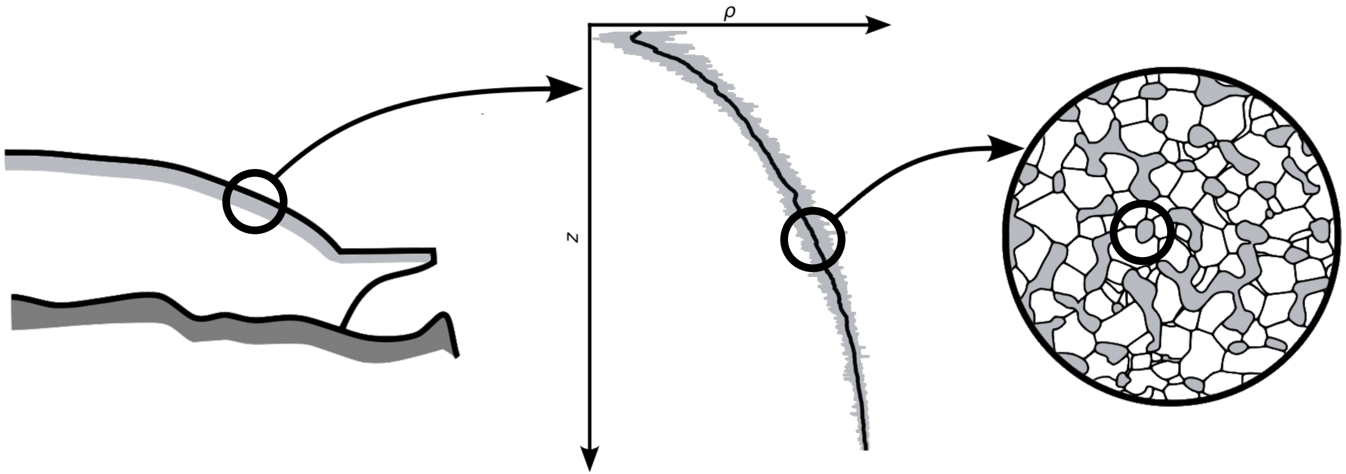

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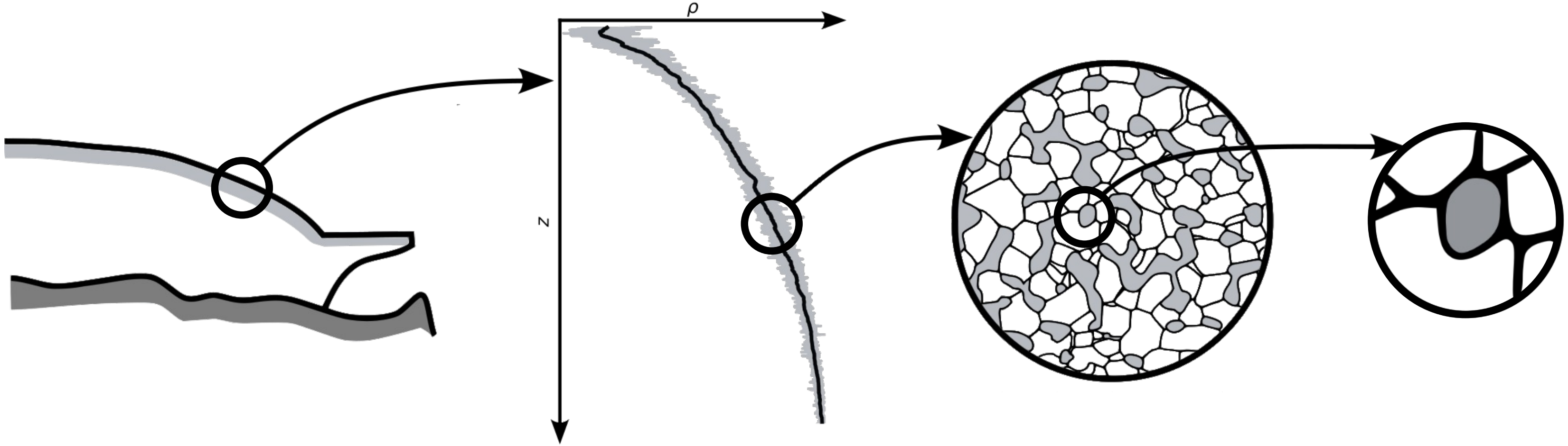

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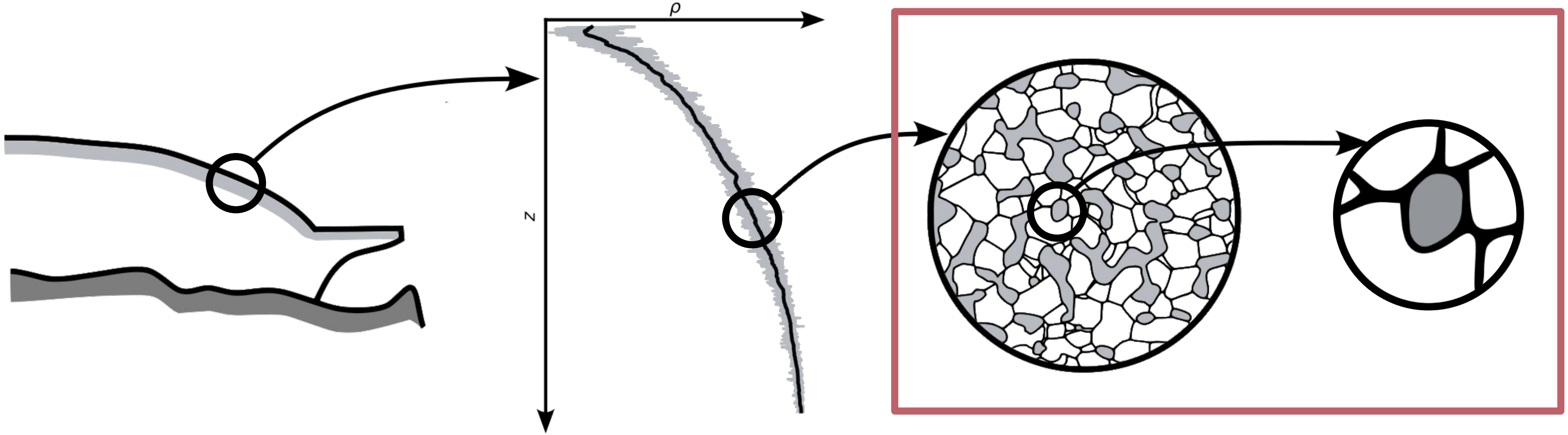

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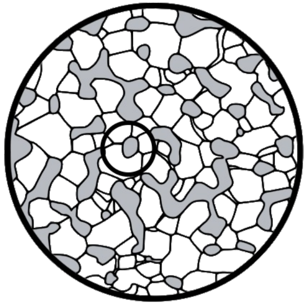

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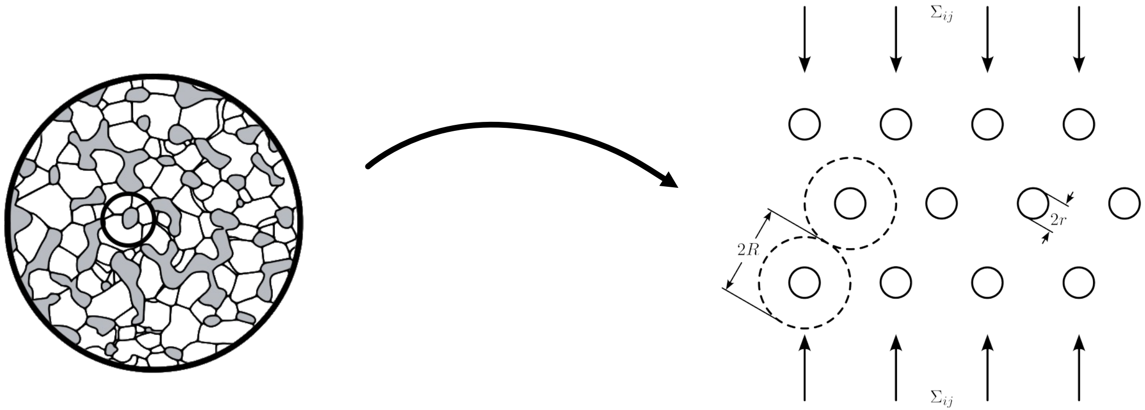

Cocks, A. C. F. (1989). *Inelastic deformation of porous materials.* Journal of the Mechanics of Solids and Physics, 37(6), pp. 693-715. https://doi.org/https://doi.org/10.1016/0022-5096(89)90014-8

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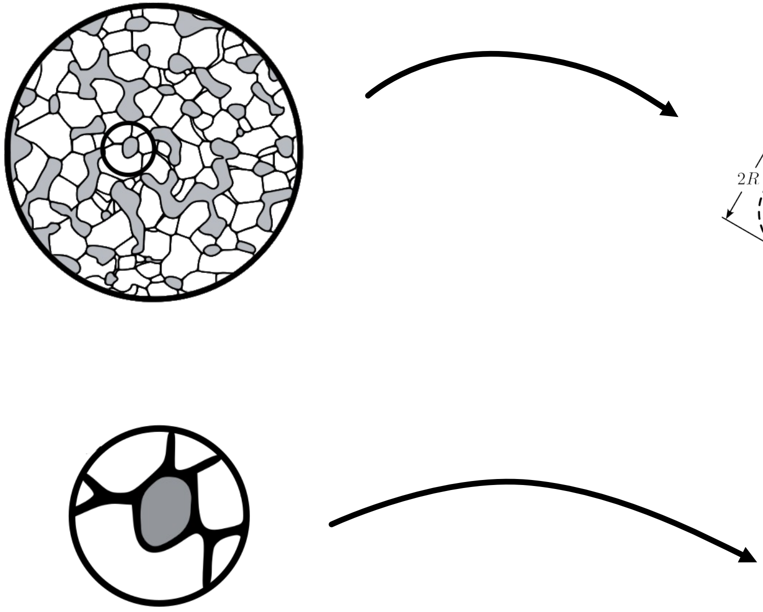

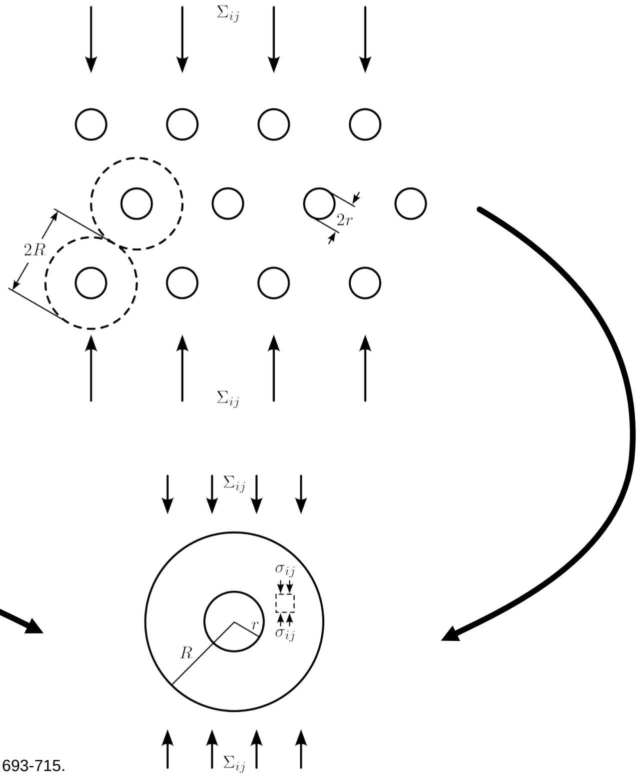

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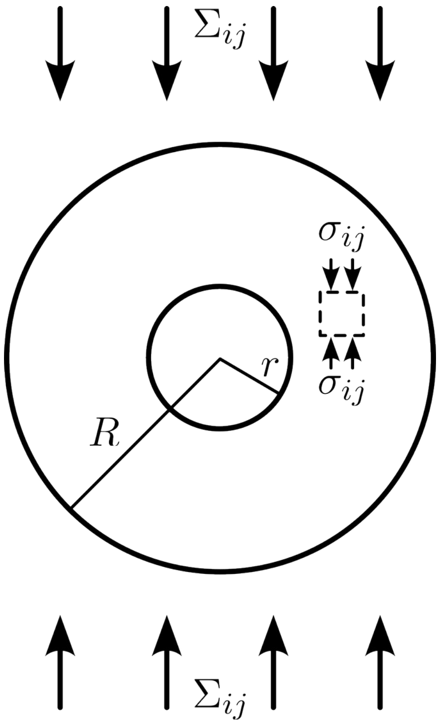

	macro-scale		**micro-scale**
Σ_{ij}	macroscopic stress tensor	σ_{ij}	microscopic stress tensor
$\dot{E}_{ij}$	macroscopic strain rate tensor	$\dot{\varepsilon}_{ij}$	microscopic strain rate tensor

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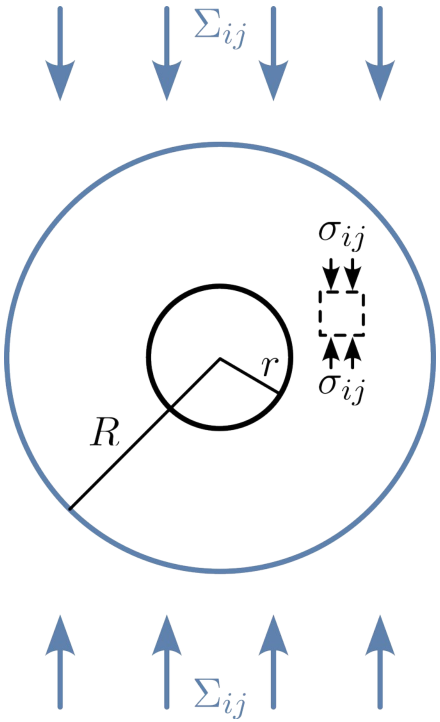

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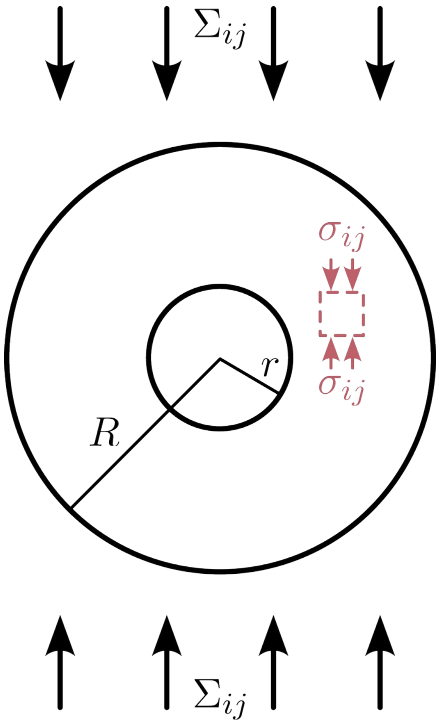

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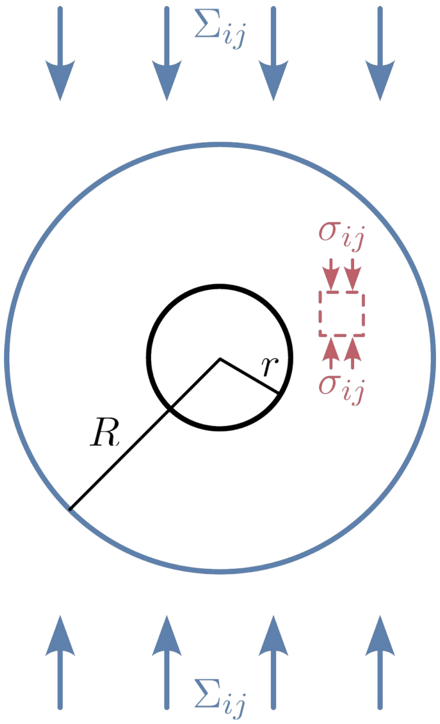

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Homogenization - The Cell Model Approach

Annals of Glaciology 24 1997

Flow simulation of a firn-covered cold glacier

Olivier Gagliardini, Jacques Meyssonnier
Laboratoire de Glaciologie et Géophysique de l'Environnement du CNRS,
associé à l'Université Joseph Fourier (UJF-Grenoble I), BP 96, 38402 Saint-Martin-d'Hères Cedex, France

Gagliardini, O. and Meyssonnier, J. (1997). *Flow simulation of a firn-covered cold glacier.* Annals of Glaciology, 24, pp. 243-248. https://doi.org/https://doi.org/10.3189/S0260305500012246

Homogenization - The Cell Model Approach

Annals of Glaciology 45 2007 29

A full Stokes-flow thermo-mechanical model for firn and ice applied to the Gorshkov crater glacier, Kamchatka

Thomas ZWINGER,[1] Ralf GREVE,[2] Olivier GAGLIARDINI,[3] Takayuki SHIRAIWA,[4]
Mikko LYLY[1]

[1]*CSC–Scientific Computing Ltd., P.O. Box 405, FIN-02101 Espoo, Finland*
E-mail: thomas.zwinger@csc.fi
[2]*Institute of Low Temperature Science, Hokkaido University, Kita-19, Nishi-8, Kita-ku, Sapporo 060–0819, Japan*
[3]*LGGE CNRS-UJF Grenoble I, BP 96, 38402 Saint Martin d'Hères, France*
[4]*Research Institute for Humanity and Nature, 457–4, Motoyama, Kamigamo, Kita-ku, Kyoto 603–8047, Japan*

Zwinger, T., Greve, R., Gagliardini, O., Shiraiwa, T., and Lyly, M. (2007). *A full Stokes-flow thermo-mechanical model for firn and ice applied to the Goshkov crater glacier, Kamchatka.* Annals of Glaciology, 45, pp. 29-37. https://doi.org/10.3189/172756407782282543

Homogenization - The Cell Model Approach

Journal of Glaciology

A full Stokes ice-flow model to assist the interpretation of millennial-scale ice cores at the high-Alpine drilling site Colle Gnifetti, Swiss/Italian Alps

Carlo Licciulli[1,2], Pascal Bohleber[2,3,†], Josef Lier[2,3], Olivier Gagliardini[4], Martin Hoelzle[5] and Olaf Eisen[6,7]

Licciulli, C., Bohleber, P., Lier, J., Gagliardini, O., Hoelzle, M., and Eisen, O. (2020). *A full Stokes ice-flow model to assist the interpretation of millenial-scale ice cores at the high-Alpine drilling site Colle Gnifetti, Swiss/Italian Alps.* Journal of Glaciology, 66(255), pp. 35-48. https://doi.org/10.1017/jog.2019.82

Homogenization - The Cell Model Approach

Budiansky, B., Hutchinson, J. W., and Slutsky, S. (1982). *Void Growth and Collapse in Viscous Solids.* Mechanics of Solids, The Rodney Hill 60th Anniversary Volume, pp. 13-45. https://doi.org/10.1016/B978-0-08-025443-2.50009-4

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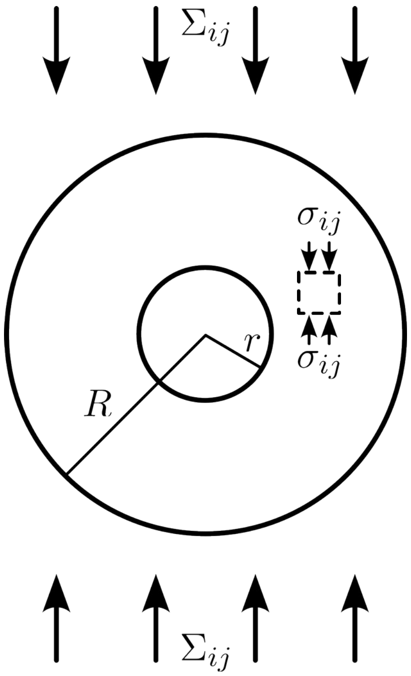
Duva, J. M. and Crow, P. D. (1992). *The densification of powders by power-law creep during hot isostatic pressing.* Acta Metallurgica et Materialia, 40(1), pp. 31-34. https://doi.org/10.1016/0956-7151(92)90196-L

Duva, J. M. and Crow, P. D. (1994). *Analysis of consolidation of reinforced materials by power-law creep.* Mechanics of Materials, 17(1), pp. 25-32. https://doi.org/10.1016/0167-6636(94)90011-6

Homogenization - The Cell Model Approach



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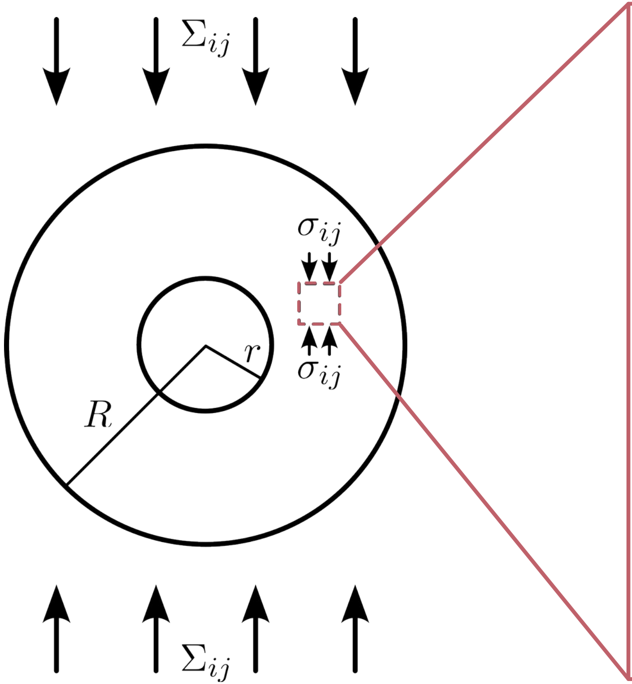

$\dot{\varepsilon}_e, \ \tilde{\Phi}$

Glen's Flow Law (Glen, 1955)

$$\dot{\varepsilon}_e = B\sigma_e^n$$

(Nye's) generalization of Glen's Flow Law (Nye, 1957)

$$\dot{e}_{ij} = \frac{3}{2} B\sigma_e^{(n-1)} s_{ij}$$

flow potential

$$\tilde{\Phi} = \frac{1}{(n+1)} B^* \sigma_e^{(n+1)}$$

Glen, J. W. (1955). *The creep of polycrystalline ice.* Proceedings of the Royal Society A, 228(1175), pp. 519-538. https://doi.org/10.1098/rspa.1955.0066
Nye, J. F. (1957). *The distribution of stress and velocity in glaciers and ice-sheets.* Proceedings of the Royal Society A, 238(1216), pp. 113-133. https://doi.org/10.1098/rspa.1957.0026
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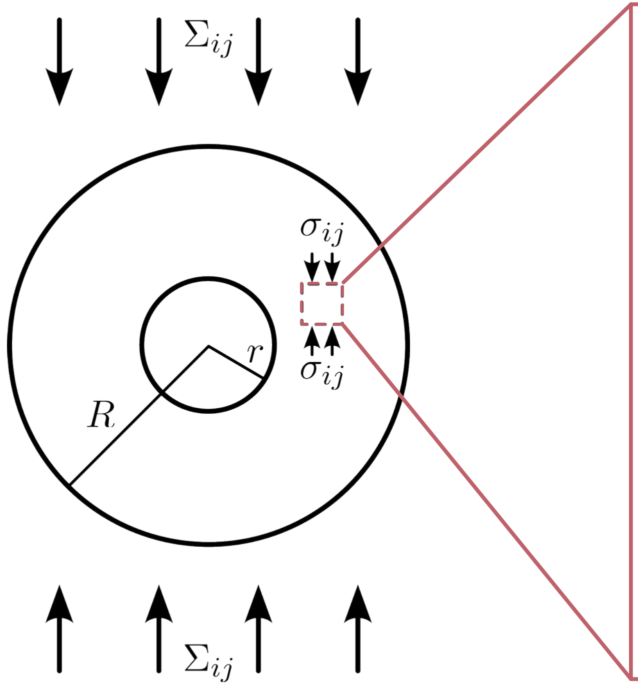
Homogenization - Hill Condition

Homogenization - The Cell Model Approach



σ_e, Φ

Glen's Flow Law (Glen, 1955)

$$\sigma_e = B^{-\frac{1}{n}} \dot{\varepsilon}_e^{\frac{1}{n}}$$

(Nye's) generalization of Glen's Flow Law (Nye, 1957)

$$\dot{e}_{ij} = \frac{3}{2} B^{\frac{1}{n}} \dot{\varepsilon}_e^{\frac{(n-1)}{n}} s_{ij}$$

strain energy rate density

$$\Phi = \frac{n}{(n+1)} B^{*-\frac{1}{n}} \dot{\varepsilon}_e^{\frac{(n+1)}{n}}$$

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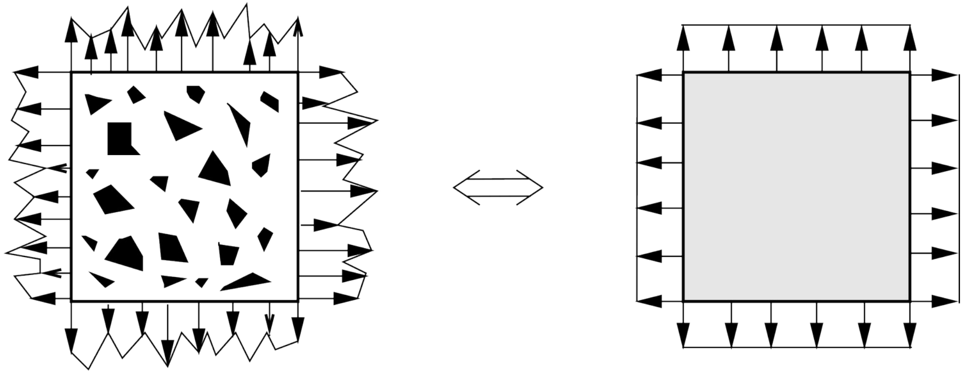


Representative volume element with fluctuating microscopic fields and averages illustrating the Hill Condition (Gross and Seelig, 2018, p. 266).

$$\Sigma_{ij}\dot{E}_{ij} = \frac{1}{V}\int_V \sigma_{ij}\dot{\varepsilon}_{ij}\, dV$$

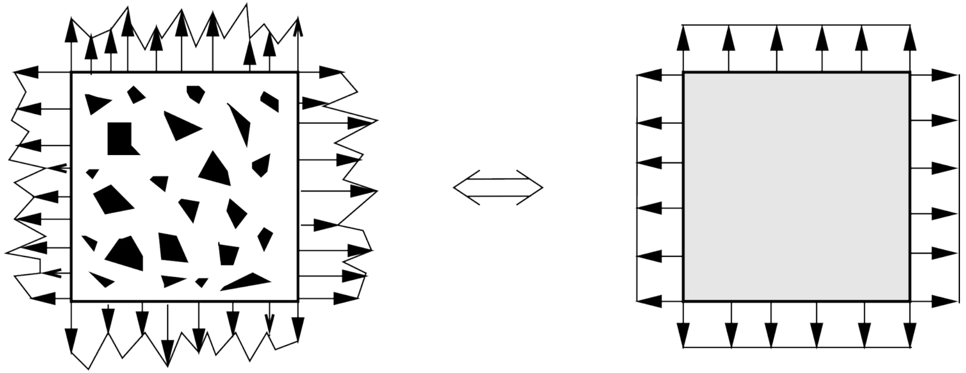
(Hill, 1963)

Hill, R. (1963). *Elastic properties of reinforced solids: Some theoretical principles.* Journal of Mechanics and Physics of Solids, 11(5), pp. 357-372. https://doi.org/10.1016/0022-5096(63)90036-Xfs

Gross, D. and Seelig, T. (2018). *Fracture Mechanics With an Introduction to Micromechanics.* Ed. by Kulack, F. A., 3rd ed. Mechanical Engineering Series. Springer. https://doi.org/10.1007/978-3-319-71090-7

Macro-Scale Flow Potential

Homogenization – Hill Condition



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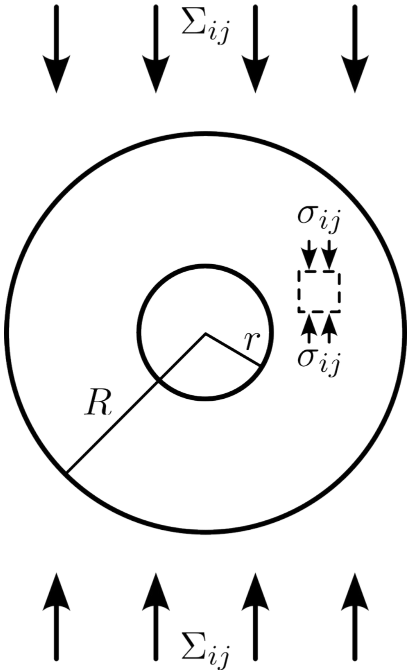
Homogenization – Macro-Scale Flow Potential



$$\Sigma_{ij}\dot{E}_{ij} = \frac{1}{V}\int_V \sigma_{ij}\dot{\varepsilon}_{ij}\, dV$$

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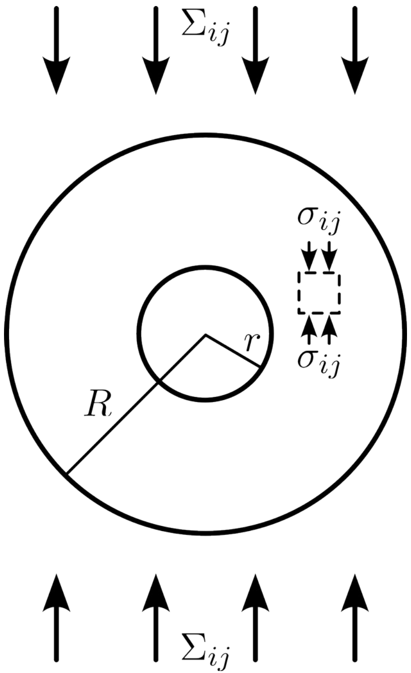
Homogenization – Macro-Scale Flow Potential



$$\Sigma_{ij}\dot{E}_{ij} = \frac{1}{V}\int_{V_m} \sigma_{ij}\dot{\varepsilon}_{ij}\, dV$$

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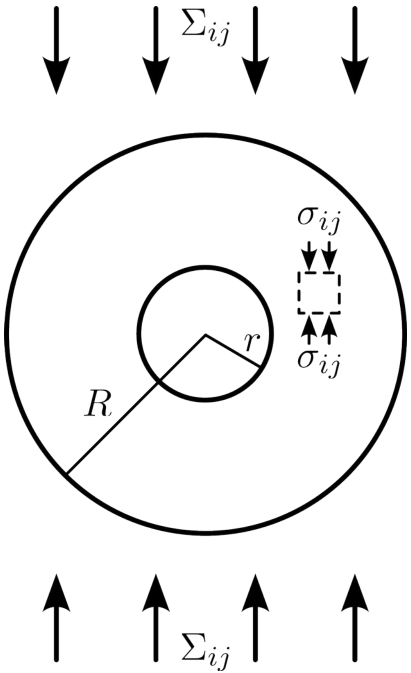
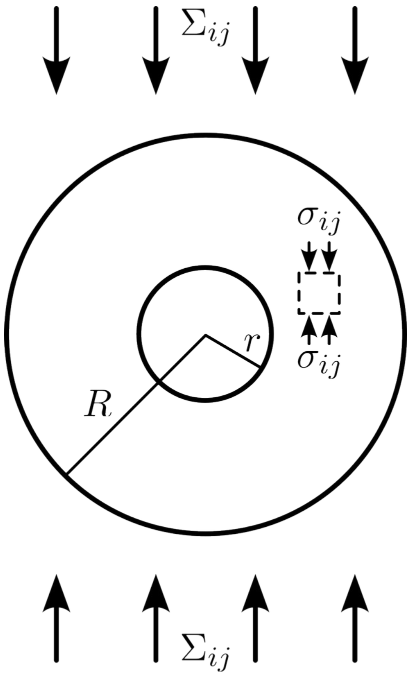
Homogenization – Macro-Scale Flow Potential



$$d\tilde{\Psi} = d\Sigma_{ij}\dot{E}_{ij} = \frac{1}{V}\int_{V_m} d\sigma_{ij}\dot{\varepsilon}_{ij}\, dV$$

Cocks, A. C. F. (1989). *Inelastic deformation of porous materials.* Journal of the Mechanics of Solids and Physics, 37(6), pp. 693-715. https://doi.org/10.1016/0022-5096(89)90014-8

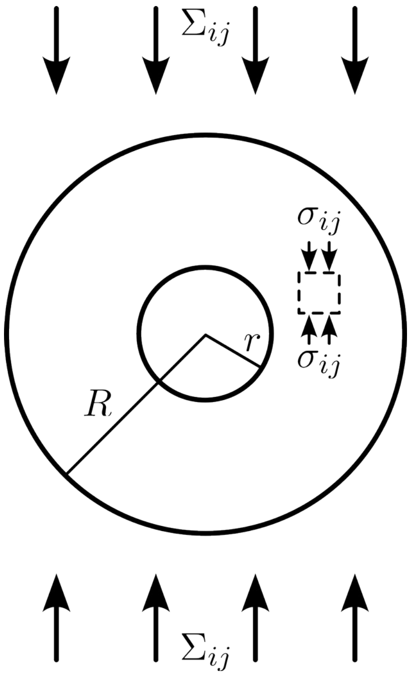
Homogenization – Macro-Scale Flow Potential



$$d\Psi = \frac{1}{V} \int_{V_m} d\tilde{\Phi} \, dV$$

Cocks, A. C. F. (1989). *Inelastic deformation of porous materials.* Journal of the Mechanics of Solids and Physics, 37(6), pp. 693-715. https://doi.org/10.1016/0022-5096(89)90014-8

Homogenization – Macro-Scale Flow Potential



$$\tilde{\Psi} = \frac{1}{V} \int_{V_m} \tilde{\Phi} \, dV$$

$$\tilde{\Phi} = \frac{1}{(n+1)} B^* \sigma_e^{(n+1)}$$

micro-scale flow potential

$$\tilde{\Psi} = \frac{1}{(n+1)} B^* S^{(n+1)}$$

macro-scale flow potential

Cocks, A. C. F. (1989). *Inelastic deformation of porous materials.* Journal of the Mechanics of Solids and Physics, 37(6), pp. 693-715. https://doi.org/10.1016/0022-5096(89)90014-8

Effective Stress

$$S^2_{\text{Cocks}} = \frac{\left(1 + \frac{2}{3}\left(1 - D\right)\right)}{D^{2n/(n+1)}} \Sigma_e^2 + \frac{\frac{9}{4}\frac{2n}{(n+1)}\frac{(1-D)}{(2-D)}}{D^{2n/(n+1)}} \Sigma_m^2$$

Cocks, A. C. F. (1989). *Inelastic deformation of porous materials.* Journal of the Mechanics of Solids and Physics, 37(6), pp. 693-715. https://doi.org/10.1016/0022-5096(89)90014-8

effective stress comparison

Effective Stress

$$S^2_{\text{Cocks}} = \frac{\left(1 + \frac{2}{3}\left(1 - D\right)\right)}{D^{2n/(n+1)}} \Sigma_e^2 + \frac{\frac{9}{4}\,\frac{2n}{(n+1)}\,\frac{(1-D)}{(2-D)}}{D^{2n/(n+1)}} \Sigma_m^2$$

Cocks, A. C. F. (1989). *Inelastic deformation of porous materials.* Journal of the Mechanics of Solids and Physics, 37(6), pp. 693-715. https://doi.org/10.1016/0022-5096(89)90014-8

$$S^2_{\text{WilkinsonAshby}} = \left(\frac{n\left(1 - D\right)}{\left(1 - \left(1 - D\right)^{1/n}\right)^n}\right)^{2/(n+1)} \left(\frac{3}{2n}\right)^2 \Sigma_m^2$$

Wilkinson, D. S. and Ashby, M. F. (1975). *Pressure sintering by power law creep.* Acta Metallurgica, 23(11), pp. 1277-1285. https://doi.org/10.1016/0001-6160(75)90136-4

effective stress comparison

Effective Stress

$$S^2_{\text{Cocks}} = \frac{\left(1 + \frac{2}{3}\left(1 - D\right)\right)}{D^{2n/(n+1)}} \Sigma_e^2 + \frac{\frac{9}{4} \frac{2n}{(n+1)} \frac{(1-D)}{(2-D)}}{D^{2n/(n+1)}} \Sigma_m^2$$

Cocks, A. C. F. (1989). *Inelastic deformation of porous materials.* Journal of the Mechanics of Solids and Physics, 37(6), pp. 693-715. https://doi.org/10.1016/0022-5096(89)90014-8

$$S^2_{\text{WilkinsonAshby}} = \left(\frac{n\left(1 - D\right)}{\left(1 - \left(1 - D\right)^{1/n}\right)^n} \right)^{2/(n+1)} \left(\frac{3}{2n}\right)^2 \Sigma_m^2$$

Wilkinson, D. S. and Ashby, M. F. (1975). *Pressure sintering by power law creep.* Acta Metallurgica, 23(11), pp. 1277-1285. https://doi.org/10.1016/0001-6160(75)90136-4

$$S^2_{\text{D\&C}} = \frac{\left(1 + \frac{2}{3}\left(1 - D\right)\right)}{D^{2n/(n+1)}} \Sigma_e^2 + \left(\frac{n\left(1 - D\right)}{\left(1 - \left(1 - D\right)^{1/n}\right)^n} \right)^{2/(n+1)} \left(\frac{3}{2n}\right)^2 \Sigma_m^2$$

effective stress comparison

Duva, J. M. and Crow, P. D. (1994). *Analysis of consolidation of reinforced materials by power-law creep.* Mechanics of Materials, 17(1), pp. 25-32. https://doi.org/10.1016/0167-6636(94)90011-6

Effective Stress

$$S^2_{\mathrm{D\&C}} = \frac{\left(1 + \frac{2}{3}\left(1-D\right)\right)}{D^{2n/(n+1)}} \Sigma_e^2 + \left(\frac{n\left(1-D\right)}{\left(1-\left(1-D\right)^{1/n}\right)^n}\right)^{2/(n+1)} \left(\frac{3}{2n}\right)^2 \Sigma_m^2$$

Duva, J. M. and Crow, P. D. (1994). *Analysis of consolidation of reinforced materials by power-law creep.* Mechanics of Materials, 17(1), pp. 25-32. https://doi.org/10.1016/0167-6636(94)90011-6

general form of the effective stress

$$S = \left(a(\rho)\,\Sigma_e^2 + b(\rho)\,\Sigma_m^2\right)^{\frac{1}{2}}$$

with

von Mises equivalent stress

$$\Sigma_e^2 = \frac{3}{2}\Sigma_{ij}^D \Sigma_{ij}^D$$

deviatoric stress tensor

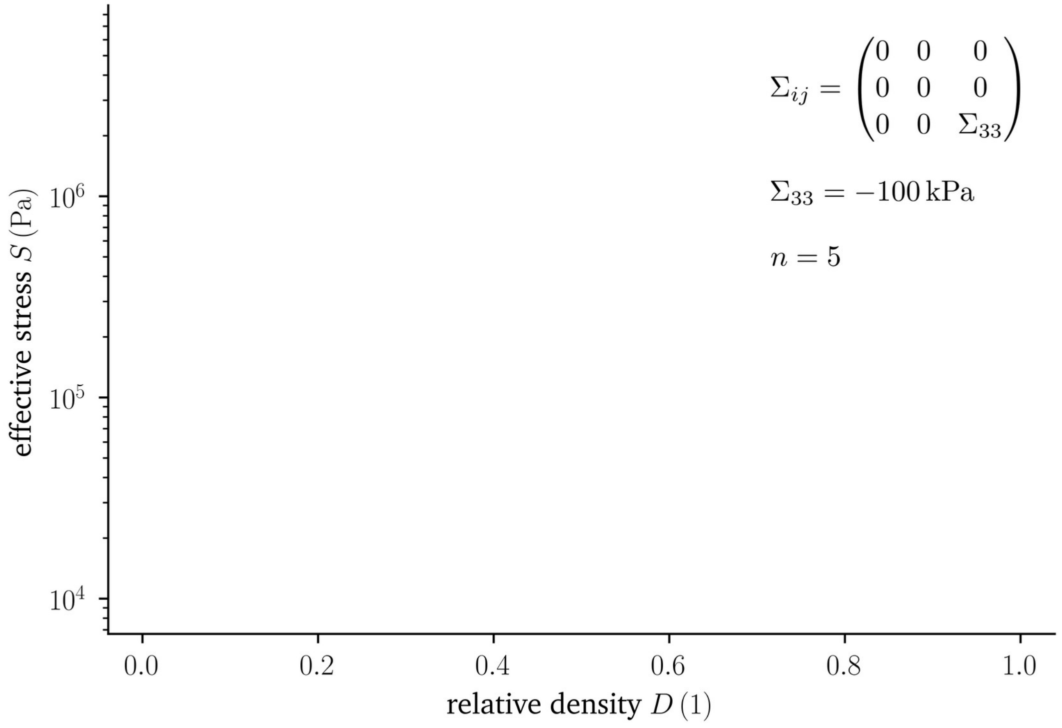
$$\Sigma_{ij}^D = \Sigma_{ij} - \frac{1}{3}\Sigma_{kk}\delta_{ij}$$

mean stress

$$\Sigma_m = \frac{1}{3}\Sigma_{kk}$$

effective stress comparison

Effective Stress

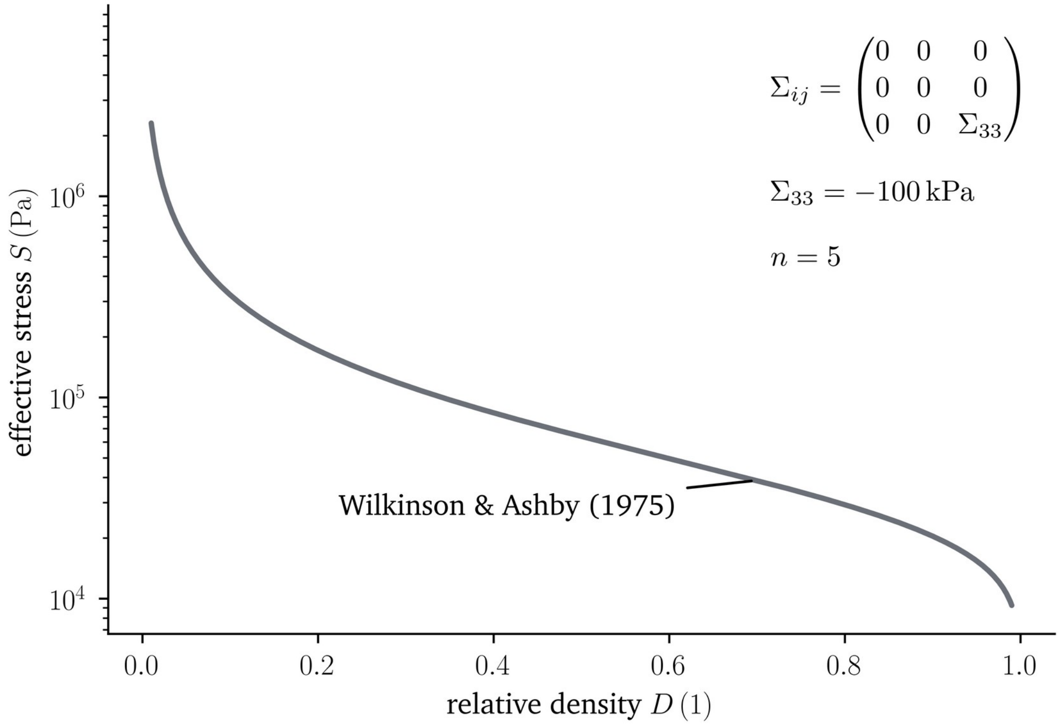


Wilkinson and Ashby (1975)

Cocks (1989)

Duva and Crow (1994)

Effective Stress



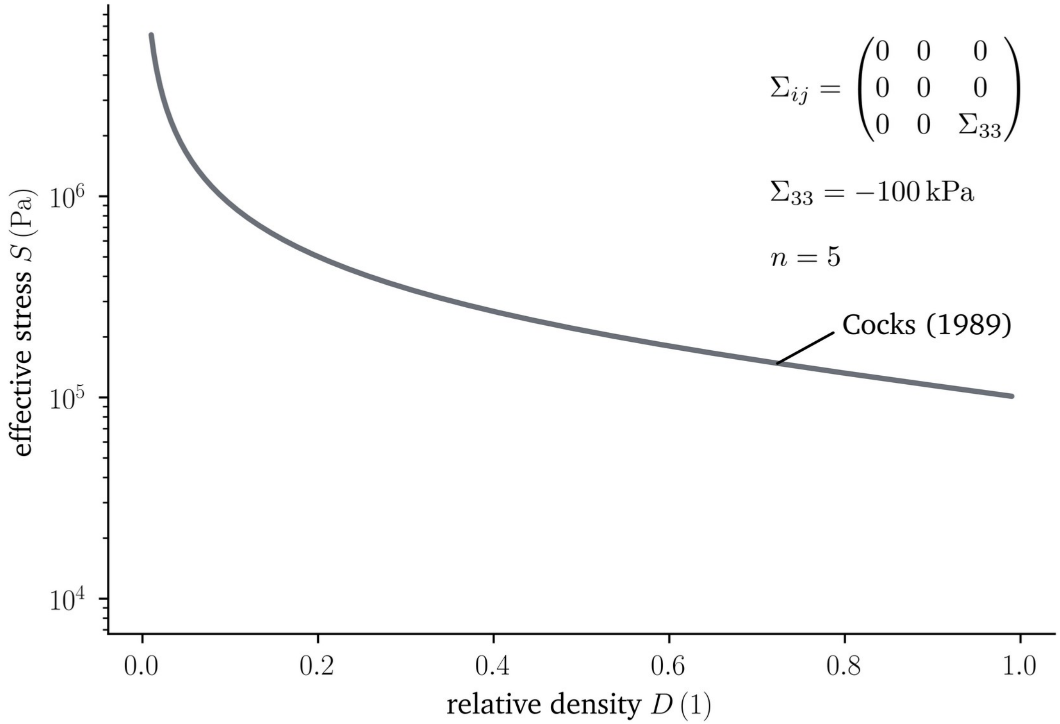
Wilkinson and Ashby (1975)

Cocks (1989)

Duva and Crow (1994)

Timm Schultz, Angelika Humbert, and Ralf Müller

Effective Stress



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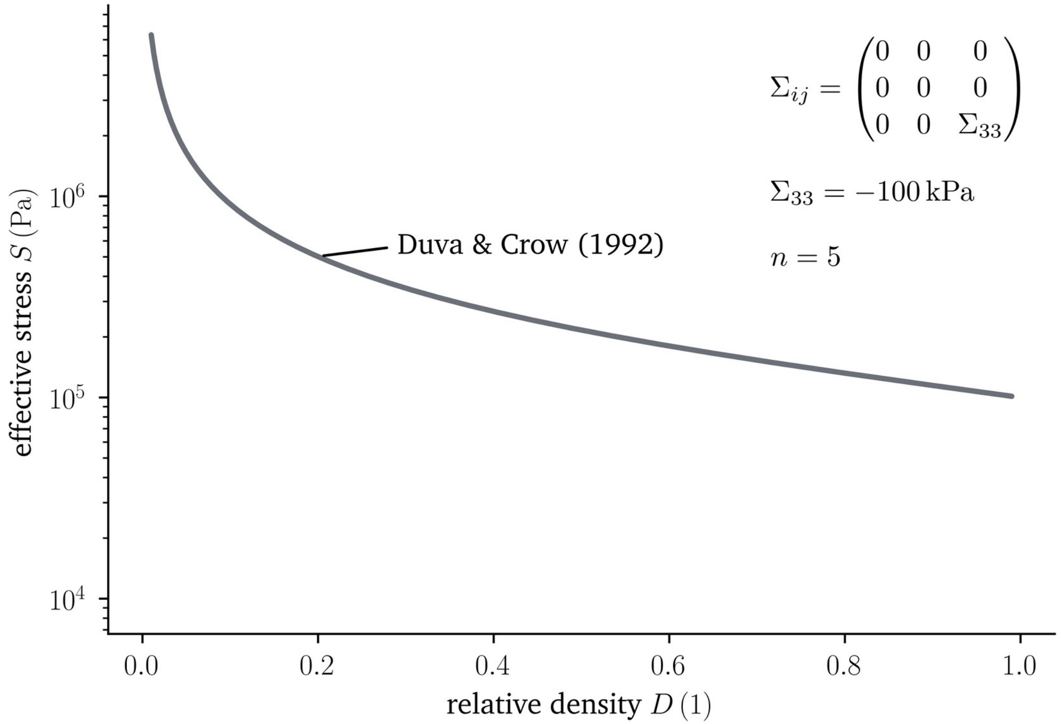
Effective Stress



Wilkinson and Ashby (1975)

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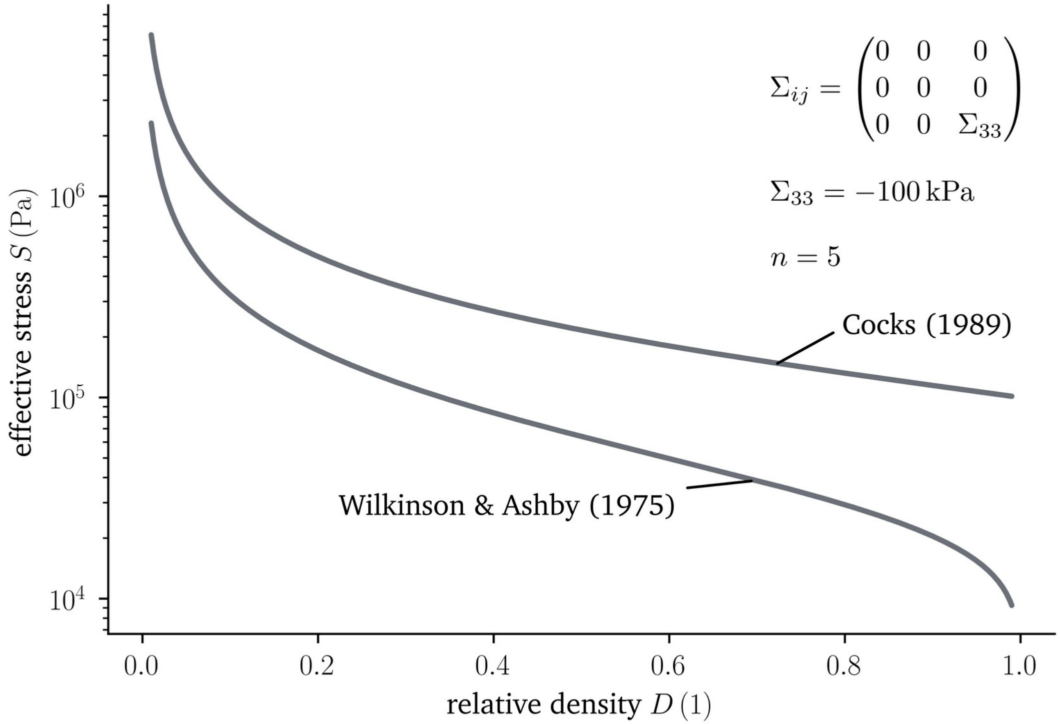
Effective Stress



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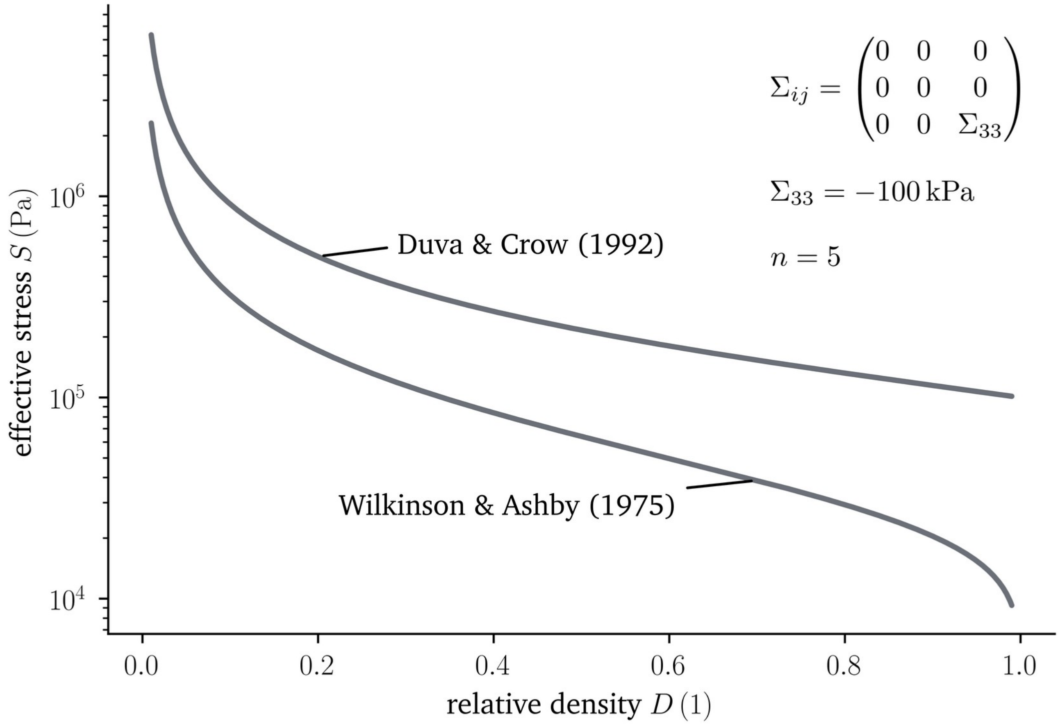
Effective Stress



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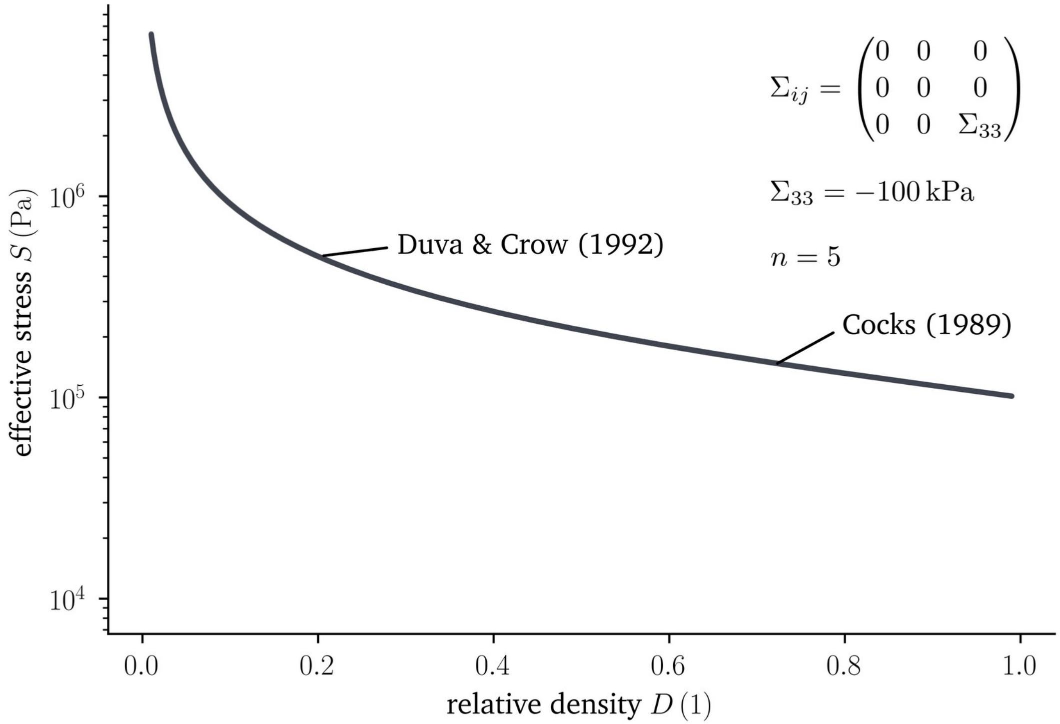
Effective Stress



Wilkinson and Ashby (1975)

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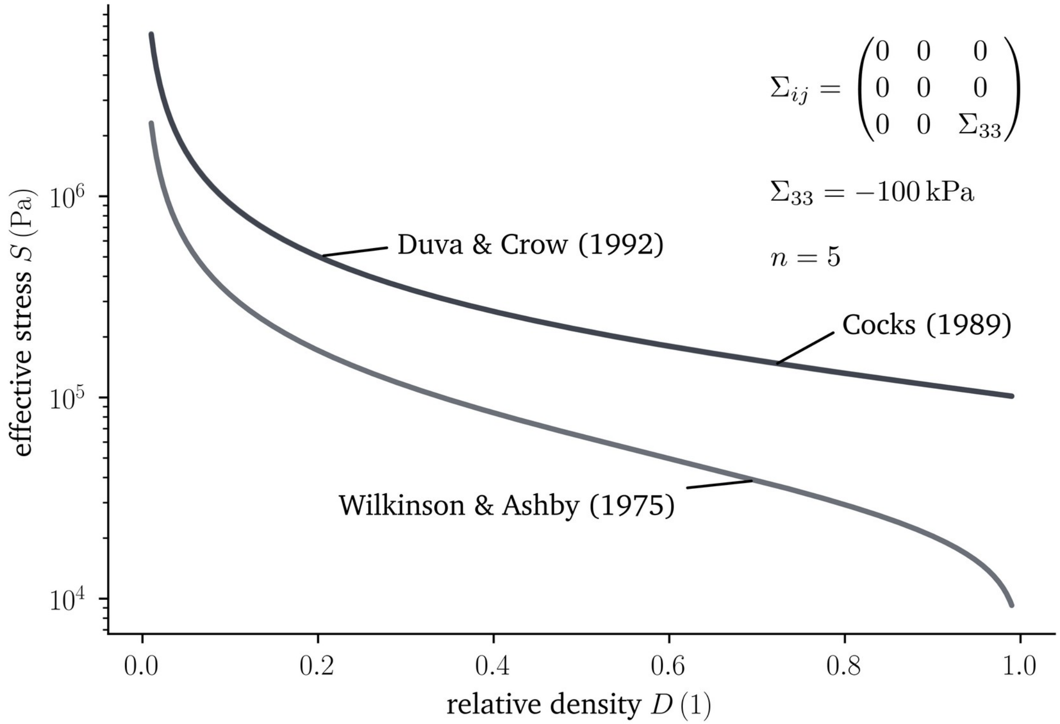
Effective Stress



Wilkinson and Ashby (1975)

Cocks (1989)

Duva and Crow (1994)

Material Model – Strain Rate Form

$$\dot{E}_{ij} = \frac{\partial \tilde{\Psi}}{\partial \Sigma_{ij}}$$

$$\tilde{\Psi} = \frac{1}{(n+1)} B^* S^{(n+1)}$$

flow potential

$$S = \left(a(\rho)\,\Sigma_e^2 + b(\rho)\,\Sigma_m^2\right)^{\frac{1}{2}}$$

effective stress

$$\dot{E}_{ij} = \frac{1}{2} B^*(T)\,\left(a(\rho)\,\Sigma_e^2 + b(\rho)\Sigma_m^2\right)^{\frac{(n-1)}{2}} \left(3a(\rho)\,\Sigma_{ij}^D + \frac{2}{3}\,b(\rho)\,\Sigma_m\right)$$

stress form

Material Model – Stress Form

$$\Sigma_{ij} = \frac{\partial \Psi}{\partial \dot{E}_{ij}}$$

$$\Psi = \frac{n}{(n+1)} B^{*\,-\frac{1}{n}} E^{\frac{(n+1)}{n}}$$

strain energy rate density

$$E = \left(\frac{1}{a(\rho)} \dot{E}_e^2 + \frac{1}{b(\rho)} \dot{E}_m^2 \right)^{\frac{1}{2}}$$

effective strain rate

$$\Sigma_{ij} = \frac{1}{2} B^*(T)^{-\frac{1}{n}} \left(\frac{1}{a(\rho)} \dot{E}_e^2 + \frac{1}{b(\rho)} \dot{E}_m^2 \right)^{\frac{(1-n)}{2n}} \left(\frac{4}{3\,a(\rho)} \dot{e}_{ij} + \frac{2}{b(\rho)} \dot{E}_m \right)$$

with $\dot{E}_e^2 = \frac{2}{3} \dot{e}_{ij} \dot{e}_{ij}$ $\dot{E}_m = \dot{E}_{kk}$ $\dot{e}_{ij} = \dot{E}_{ij} - \frac{1}{3} \dot{E}_{kk} \delta_{ij}$

strain rate form

interpretation

Material Model – Interpretation

$$\Sigma_{ij} = \frac{1}{2} B^*(T)^{-\frac{1}{n}} \left(\frac{1}{a(\rho)} \dot{E}_e^2 + \frac{1}{b(\rho)} \dot{E}_m^2 \right)^{\frac{(1-n)}{2n}} \left(\frac{4}{3\, a(\rho)} \dot{e}_{ij} + \frac{2}{b(\rho)} \dot{E}_m \right)$$

stress form

Material Model – Interpretation

$$\Sigma_{ij} = \frac{2}{3\,a(\rho)}\,B^*(T)^{-\frac{1}{n}}\,E\left(\rho,\dot{E}_{ij}\right)^{\frac{(1-n)}{n}}\;\dot{e}_{ij} + \frac{1}{b(\rho)}\,B^*(T)^{-\frac{1}{n}}\,E\left(\rho,\dot{E}_{ij}\right)^{\frac{(1-n)}{n}}\;\dot{E}_m$$

$$\boldsymbol{\sigma} = -p\,(\rho,T)\;\mathbf{1} + 2\,\eta\,(\rho,T)\;\dot{\boldsymbol{\varepsilon}}^D + (\zeta\,(\rho,T)\;\mathrm{tr}\,(\dot{\varepsilon}))\,\mathbf{1}$$

Haupt, P. (2000). *Continuum Mechanics and Theory of Mateials.* Advanced Texts in Physics. Berlin, Heidelberg: Springer-Verlag. ISBN: 978-3-662-04109-3.

Greve, R. and Blatter, H. (2009). *Dynamics of Ice Sheets and Glaciers.* Ed. by Hutter, K. Advances in Geophysical and Environmental Mechanics and Mathematics. Berlin, Heidelberg: Springer. ISBN: 978-3-642-03414-5.

stress form

Material Model – Interpretation

$$\Sigma_{ij} = \boxed{\frac{2}{3\,a(\rho)}\,B^*(T)^{-\frac{1}{n}}\,E\left(\rho,\dot{E}_{ij}\right)^{\frac{(1-n)}{n}}}\,\dot{e}_{ij} + \frac{1}{b(\rho)}\,B^*(T)^{-\frac{1}{n}}\,E\left(\rho,\dot{E}_{ij}\right)^{\frac{(1-n)}{n}}\,\dot{E}_m$$

shear viscosity

$$\boldsymbol{\sigma} = -p\,(\rho,T)\,\mathbf{1} + 2\,\eta\,(\rho,T)\,\dot{\boldsymbol{\varepsilon}}^D + (\zeta\,(\rho,T)\,\mathrm{tr}\,(\dot{\varepsilon}))\,\mathbf{1}$$

$$\eta\left(\rho,T,\dot{E}_{ij}\right) = \frac{1}{3\,a(\rho)}\,B^*(T)^{-\frac{1}{n}}\left(\frac{1}{a(\rho)}\,\dot{E}_e^2 + \frac{1}{b(\rho)}\,\dot{E}_m^2\right)^{\frac{(1-n)}{2n}}$$

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stress form

Material Model – Interpretation

$$\Sigma_{ij} = \frac{2}{3\,a(\rho)} B^*(T)^{-\frac{1}{n}} E\left(\rho, \dot{E}_{ij}\right)^{\frac{(1-n)}{n}} \dot{e}_{ij} + \boxed{\frac{1}{b(\rho)} B^*(T)^{-\frac{1}{n}} E\left(\rho, \dot{E}_{ij}\right)^{\frac{(1-n)}{n}}} \dot{E}_m$$

bulk viscosity

$$\boldsymbol{\sigma} = -p\,(\rho, T)\;\mathbf{1} + 2\,\eta\,(\rho, T)\;\dot{\boldsymbol{\varepsilon}}^D + (\zeta\,(\rho, T)\;\mathrm{tr}\,(\dot{\varepsilon}))\,\mathbf{1}$$

$$\zeta\left(\rho, T, \dot{E}_{ij}\right) = \frac{1}{b(\rho)} B^*(T)^{-\frac{1}{n}} \left(\frac{1}{a(\rho)}\dot{E}_e^2 + \frac{1}{b(\rho)}\dot{E}_m^2\right)^{\frac{(1-n)}{2n}}$$

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stress form

Material Model – Interpretation

$$\Sigma_{ij} = \boxed{\frac{2}{3\,a(\rho)}\,B^*(T)^{-\frac{1}{n}}\,E\left(\rho,\dot{E}_{ij}\right)^{\frac{(1-n)}{n}}}\,\dot{e}_{ij} + \boxed{\frac{1}{b(\rho)}\,B^*(T)^{-\frac{1}{n}}\,E\left(\rho,\dot{E}_{ij}\right)^{\frac{(1-n)}{n}}}\,\dot{E}_m$$

shear viscosity

bulk viscosity

$$\boldsymbol{\sigma} = -p\,(\rho,T)\,\mathbf{1} + 2\,\eta\,(\rho,T)\,\dot{\boldsymbol{\varepsilon}}^D + (\zeta\,(\rho,T)\,\mathrm{tr}\,(\dot{\varepsilon}))\,\mathbf{1}$$

$$\eta\left(\rho,T,\dot{E}_{ij}\right) = \frac{1}{3\,a(\rho)}\,B^*(T)^{-\frac{1}{n}}\left(\frac{1}{a(\rho)}\,\dot{E}_e^2 + \frac{1}{b(\rho)}\,\dot{E}_m^2\right)^{\frac{(1-n)}{2n}}$$

$$\zeta\left(\rho,T,\dot{E}_{ij}\right) = \frac{1}{b(\rho)}\,B^*(T)^{-\frac{1}{n}}\left(\frac{1}{a(\rho)}\,\dot{E}_e^2 + \frac{1}{b(\rho)}\,\dot{E}_m^2\right)^{\frac{(1-n)}{2n}}$$

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stress form

Model Inversion – Constitutive Equation

$$\Sigma_{ij} = \frac{2}{3\,a(\rho)}\,B^*(T)^{-\frac{1}{n}}\,E\left(\rho,\dot{E}_{ij}\right)^{\frac{(1-n)}{n}}\;\dot{e}_{ij} + \frac{1}{b(\rho)}\,B^*(T)^{-\frac{1}{n}}\,E\left(\rho,\dot{E}_{ij}\right)^{\frac{(1-n)}{n}}\;\dot{E}_m$$

Model Inversion – Core Data

Timm Schultz, Angelika Humbert, and Ralf Müller

Model Inversion – Constitutive Equation

$$\frac{1}{2}B^*(T)^{-\frac{1}{n}}\left(\frac{1}{a_{\text{firn}}(\rho)}\dot{E}_e^2+\frac{1}{b_{\text{firn}}(\rho)}\dot{E}_m^2\right)^{\frac{(1-n)}{2n}}\left(\frac{4}{3a_{\text{firn}}(\rho)}\dot{e}_{zz}+\frac{2}{b_{\text{firn}}(\rho)}\dot{E}_m\right)-\Sigma_{zz}=0$$

to be determined

a_{firn}	parameter a
b_{firn}	parameter b

unknowns

$\dot{E}_{ij}$	strain rate tensor
Σ_{zz}	vertical stress
T	temperature
ρ	density

Model Inversion – Core Data

Model Inversion – Constitutive Equation

$$\frac{1}{2}B^*(T)^{-\frac{1}{n}}\left(\frac{1}{a_{\text{firn}}(\rho)}\dot{E}_e^2+\frac{1}{b_{\text{firn}}(\rho)}\dot{E}_m^2\right)^{\frac{(1-n)}{2n}}\left(\frac{4}{3a_{\text{firn}}(\rho)}\dot{e}_{zz}+\frac{2}{b_{\text{firn}}(\rho)}\dot{E}_m\right)-\Sigma_{zz}=0$$

to be determined

- a_{firn} parameter a
- b_{firn} parameter b

unknowns

- $\dot{E}_{ij}$ strain rate tensor
- Σ_{zz} vertical stress
- T temperature
- ρ density

Model Inversion – Core Data

Model Inversion – Constitutive Equation

$$\frac{1}{2}B^*(T)^{-\frac{1}{n}}\left(\frac{1}{a_{\text{firn}}(\rho)}\dot{E}_e^2+\frac{1}{b_{\text{firn}}(\rho)}\dot{E}_m^2\right)^{\frac{(1-n)}{2n}}\left(\frac{4}{3a_{\text{firn}}(\rho)}\dot{e}_{zz}+\frac{2}{b_{\text{firn}}(\rho)}\dot{E}_m\right)-\Sigma_{zz}=0$$

to be determined

a_{firn} parameter a

b_{firn} parameter b

unknowns

$\dot{E}_{ij}$ strain rate tensor

Σ_{zz} vertical stress

T temperature

ρ density

Model Inversion – Core Data

Model Inversion – Constitutive Equation

$$\frac{1}{2}B^*(T)^{-\frac{1}{n}}\left(\frac{1}{a_{\mathrm{firn}}(\rho)}\dot{E}_e^2+\frac{1}{b_{\mathrm{firn}}(\rho)}\dot{E}_m^2\right)^{\frac{(1-n)}{2n}}\left(\frac{4}{3a_{\mathrm{firn}}(\rho)}\dot{e}_{zz}+\frac{2}{b_{\mathrm{firn}}(\rho)}\dot{E}_m\right)-\Sigma_{zz}=0$$

to be determined

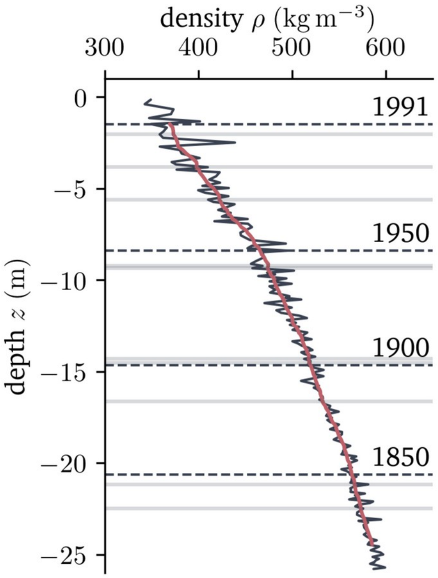
- a_{firn} parameter a
- b_{firn} parameter b

unknowns

- $\dot{E}_{ij}$ strain rate tensor
- Σ_{zz} vertical stress
- T temperature
- ρ density

Model Inversion – Core Data

Model Inversion – Core Data

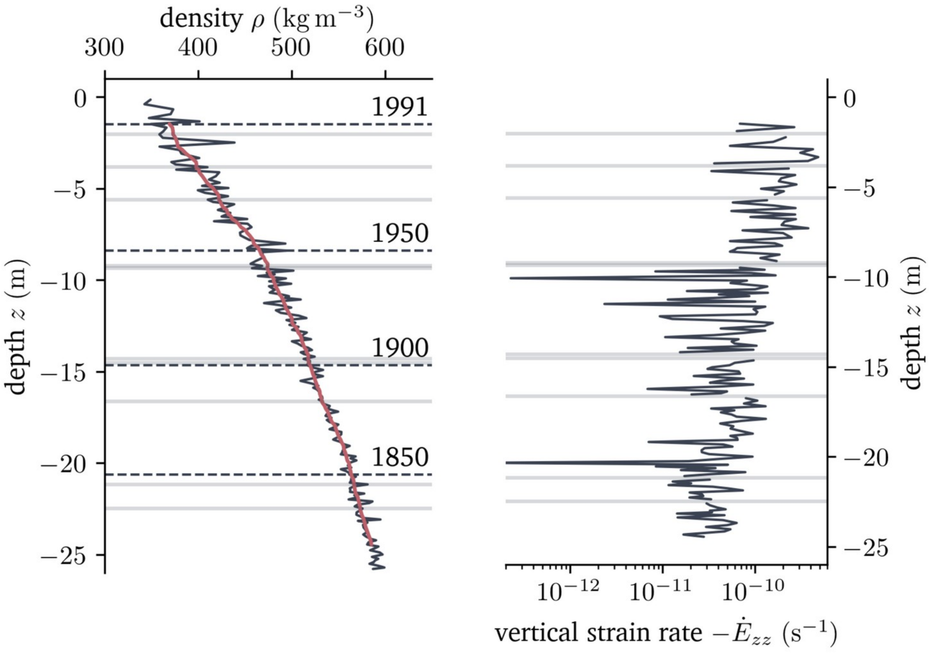


local form of the mass balance

$$\frac{\partial \rho}{\partial t} = -\rho \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right)$$

Oerter, H. F., Wilhelms, F., Jung-Rothehäusler, F., Götkas, F., Miller, H., Graf, W., and Sommer, S. (2000). *Physical Properties of firn core DML05C98_07.* PANGAEA.
Graf, W., Oerter, H., Reinwarth, O., Stichler, W., Wilhelms, F., Miller, H., and Mulvaney, R. (2002). *Calculated annual mean of d18O of firn core DML05C98_07*. PANGAEA. https://doi.org/https://doi.org/10.1594/PANGAEA.104880

Model Inversion – Core Data



local form of the mass balance

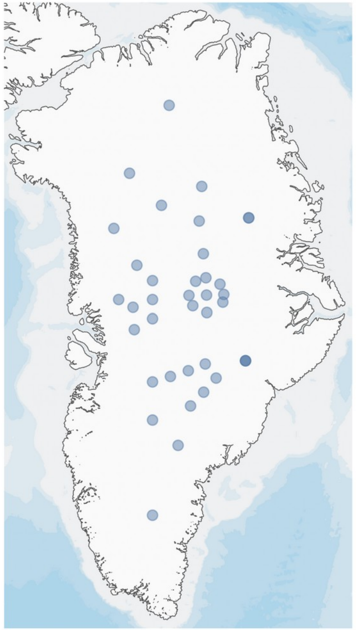
$$\frac{\partial \rho}{\partial t} = -\rho \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right)$$

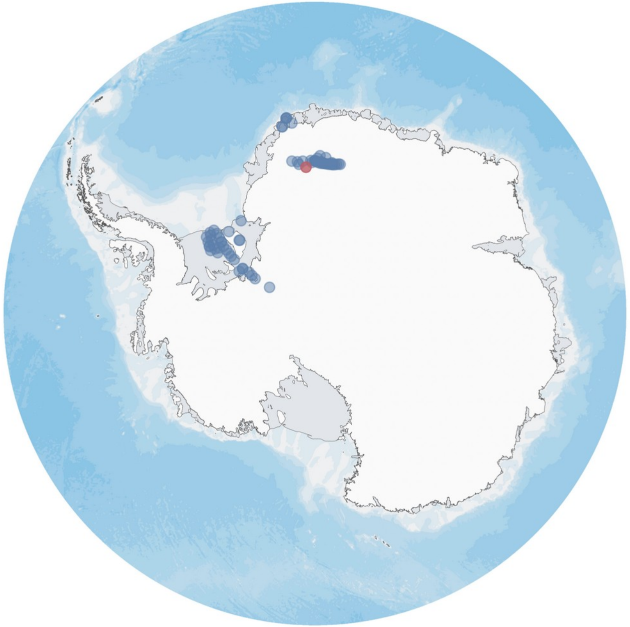
$$\dot{E}_{zz} = -\frac{1}{\rho} \frac{\partial \rho}{\partial t} - \dot{E}_{xx} - \dot{E}_{yy}$$

$$\dot{E}^{n}_{zz\,i} = -\frac{1}{\rho^{n}_{i}} \frac{\left(\rho^{n+1}_{i+1} - \rho^{n}_{i}\right)}{\left(\chi^{n+1}_{i+1} - \chi^{n}_{i}\right)} - \dot{E}_{xx} - \dot{E}_{yy}$$

Oerter, H. F., Wilhelms, F., Jung-Rothehäusler, F., Götkas, F., Miller, H., Graf, W., and Sommer, S. (2000). *Physical Properties of firn core DML05C98_07.* PANGAEA.
Graf, W., Oerter, H., Reinwarth, O., Stichler, W., Wilhelms, F., Miller, H., and Mulvaney, R. (2002). *Calculated annual mean of d18O of firn core DML05C98_07*. PANGAEA. https://doi.org/https://doi.org/10.1594/PANGAEA.104880

Model Inversion – Core Data



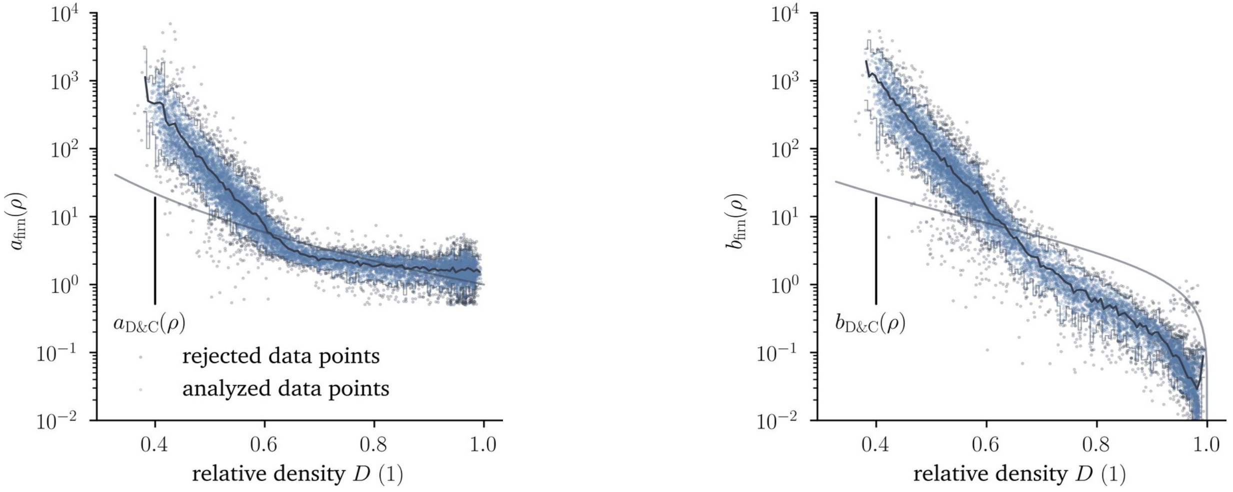


Greenland: 38 firn cores

Antarctica: 52 firn cores

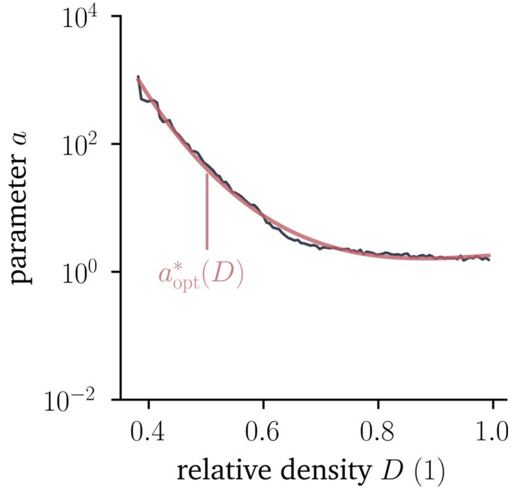
Amante, C. and Eakins, B. W. (2009). *ETOPO1 1 Arc-Minute Global Relief Model: Procedures Data Source, and Analysis.* NOAA Technical Memorandum NESDIS NGDC 24. National Geophysical Data Center, NOAA. https://repository.library.noaa.gov/view/noaa/1163
Arndt, J. E., Schenke, H. W., Jakobsson, M., Nitsche, F. O., Buys, G., Goleby, B., Rebesco, M., Bohoyo, F., Hong, J., Black, J., Greku, R., Udintsev, G., Brrios, F., Reynoso-Peralta, W., Taisei, M., and Wigley, R. (2013). *The International Bathymetric Chart of the Southern Ocean (IBSCO) Version 1.0 – A new bathymetric compilation covering circum-Antarctic waters.* Geophysical Research Letters, 40(12). pp. 3111-3117. https://doi.org/https://doi.org/10.1002/grl.50413

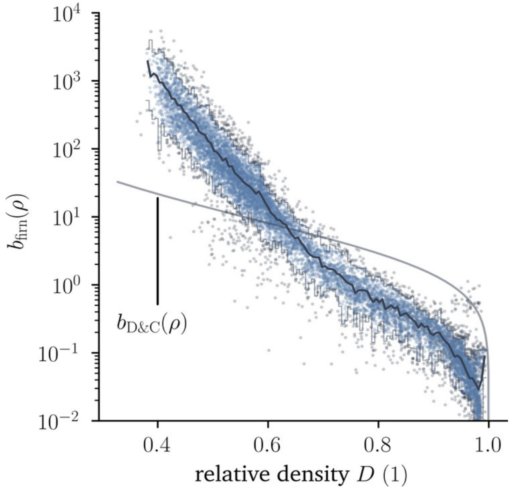
Model Inversion – Parameter Determination



Timm Schultz, Angelika Humbert, and Ralf Müller

Model Inversion – Parameter Determination

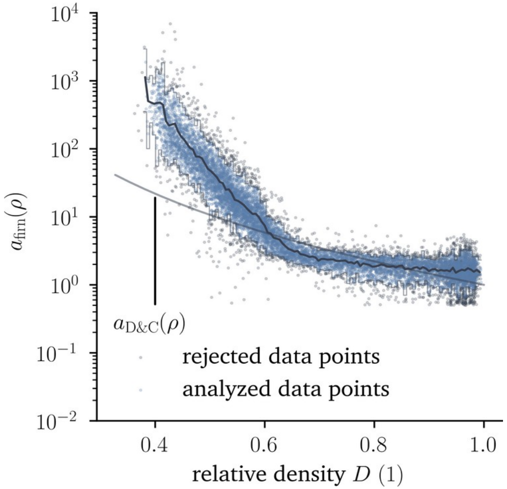


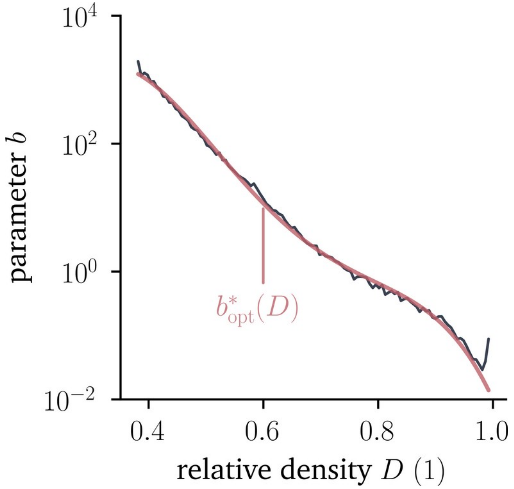


$$\log_{10}\left(a^*_{\text{opt}}\left(D\right)\right) = a_0\,D^3 + a_1\,D^2 + a_2\,D + a_3$$

$$a_0 = -12.60, \quad a_1 = 38.26, \quad a_2 = -38.06, \quad a_3 = 12.66$$

Model Inversion – Parameter Determination

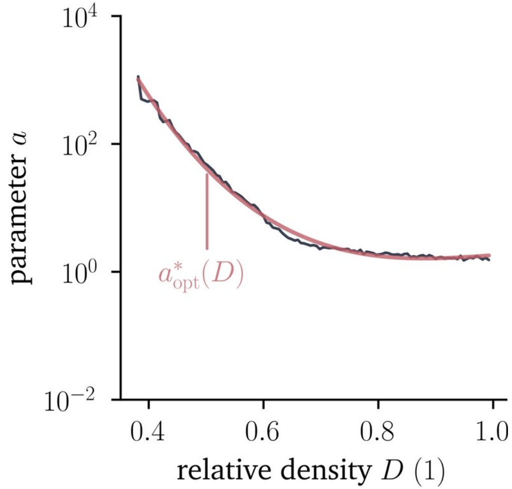


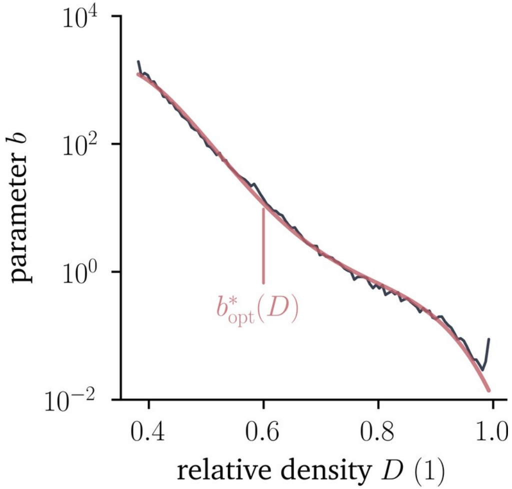


$$\log_{10}\left(b^*_{\mathrm{opt}}(D)\right) = b_0\left(D^4 - 1\right) + b_1\left(D^3 - 1\right) + b_2\left(D^2 - 1\right) + b_3\left(D - 1\right) - 2$$

$$b_0 = -142.96, \quad b_1 = 374.45, \quad b_2 = -351.10, \quad b_3 = 131.26$$

Model Inversion – Parameter Determination





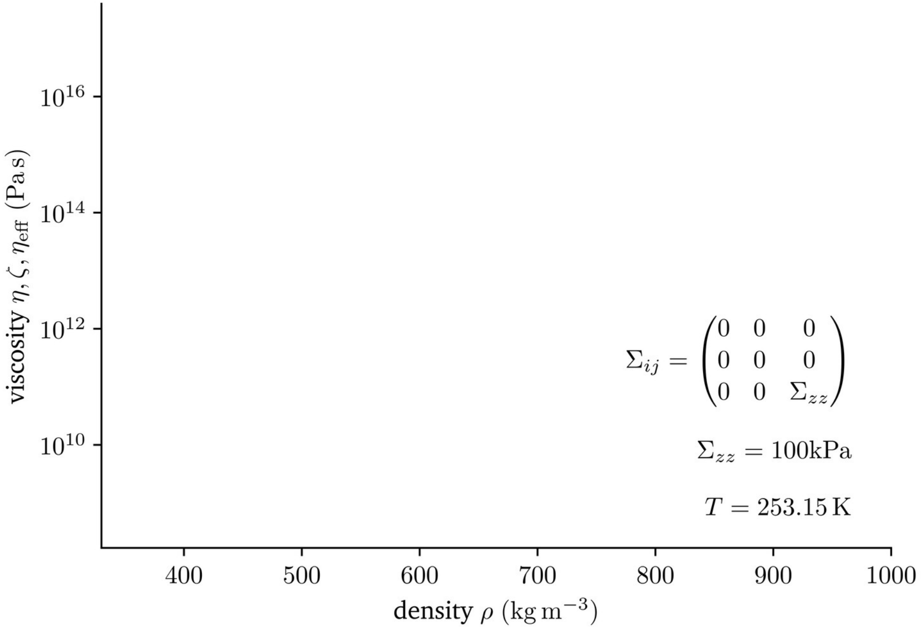
$$\log_{10}\left(a^*_{\mathrm{opt}}\left(D\right)\right) = a_0\, D^3 + a_1\, D^2 + a_2\, D + a_3$$

$$\log_{10}\left(b^*_{\mathrm{opt}}\left(D\right)\right) = b_0\,\left(D^4 - 1\right) + b_1\,\left(D^3 - 1\right) + b_2\,\left(D^2 - 1\right) + b_3\,\left(D - 1\right) - 2$$

$a_0 = -12.60, \quad a_1 = 38.26, \quad a_2 = -38.06, \quad a_3 = 12.66$

$b_0 = -142.96, \quad b_1 = 374.45, \quad b_2 = -351.10, \quad b_3 = 131.26$

Resulting Material Behavior

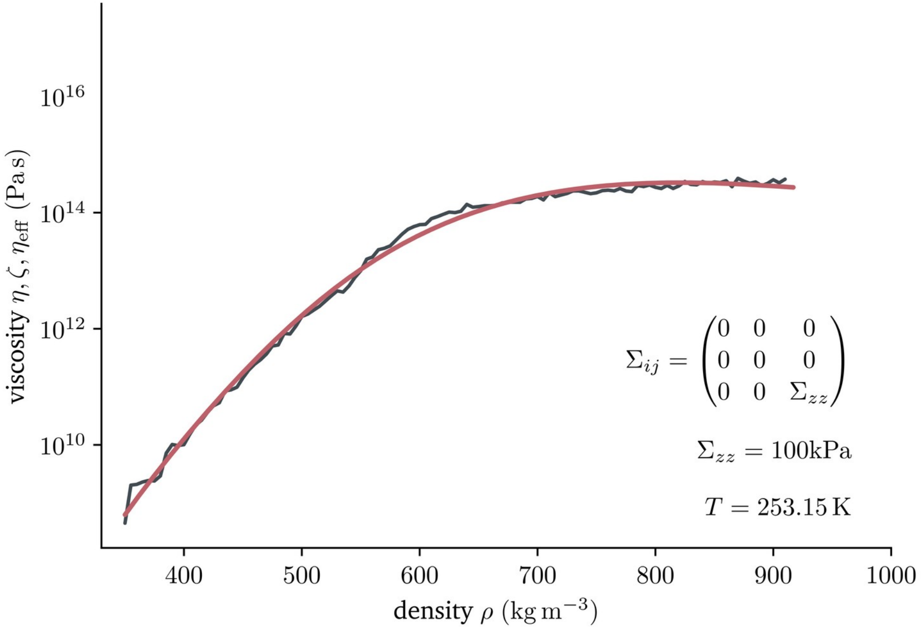


$$\eta\left(\rho, T, \Sigma_{ij}\right) = \frac{1}{3\,a(\rho)} B^*(T)^{-1} \left(a(\rho)\,\Sigma_e^2 + b(\rho)\,\Sigma_m^2\right)^{\frac{(1-n)}{2}}$$

$$\zeta\left(\rho, T, \Sigma_{ij}\right) = \frac{1}{b(\rho)} B^*(T)^{-1} \left(a(\rho)\,\Sigma_e^2 + b(\rho)\,\Sigma_m^2\right)^{\frac{(1-n)}{2}}$$

η_{ice}

Resulting Material Behavior



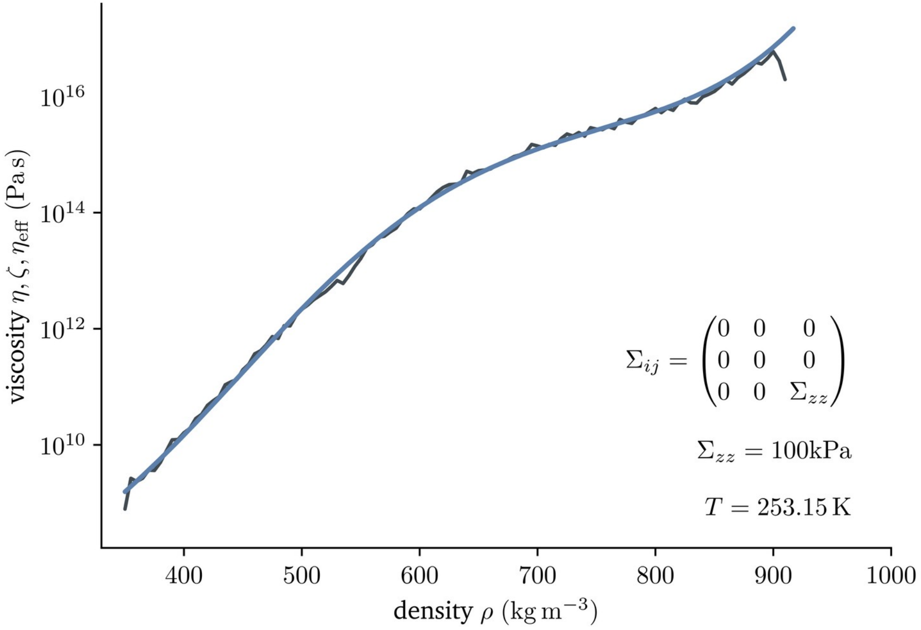
$$\eta\left(\rho, T, \Sigma_{ij}\right) = \frac{1}{3\,a(\rho)} B^*(T)^{-1} \left(a(\rho)\,\Sigma_e^2 + b(\rho)\,\Sigma_m^2\right)^{\frac{(1-n)}{2}}$$

$$\zeta\left(\rho, T, \Sigma_{ij}\right) = \frac{1}{b(\rho)} B^*(T)^{-1} \left(a(\rho)\,\Sigma_e^2 + b(\rho)\,\Sigma_m^2\right)^{\frac{(1-n)}{2}}$$

η_{ice}

Timm Schultz, Angelika Humbert, and Ralf Müller

Resulting Material Behavior



$$\eta\,(\rho, T, \Sigma_{ij}) = \frac{1}{3\,a(\rho)} B^*(T)^{-1} \left(a(\rho)\,\Sigma_e^2 + b(\rho)\,\Sigma_m^2\right)^{\frac{(1-n)}{2}}$$

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η_{ice}

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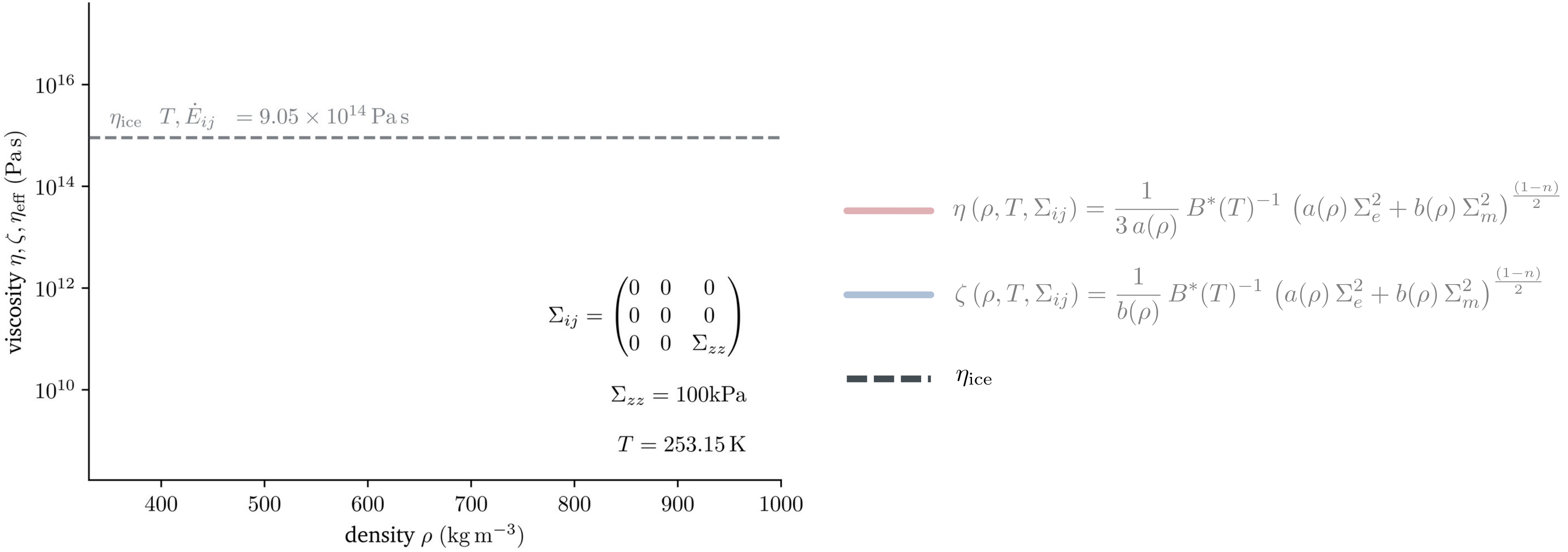
Resulting Material Behavior



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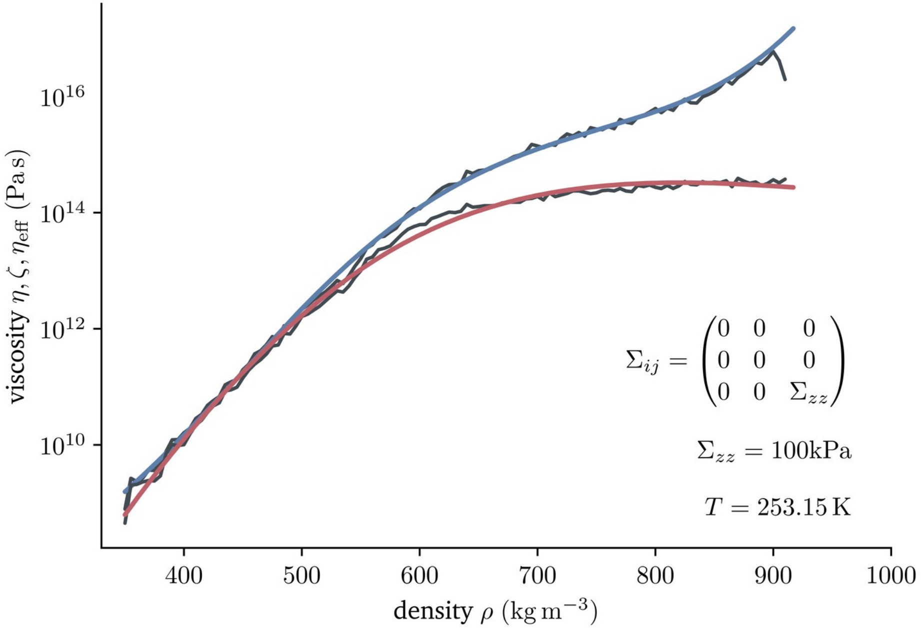
Resulting Material Behavior

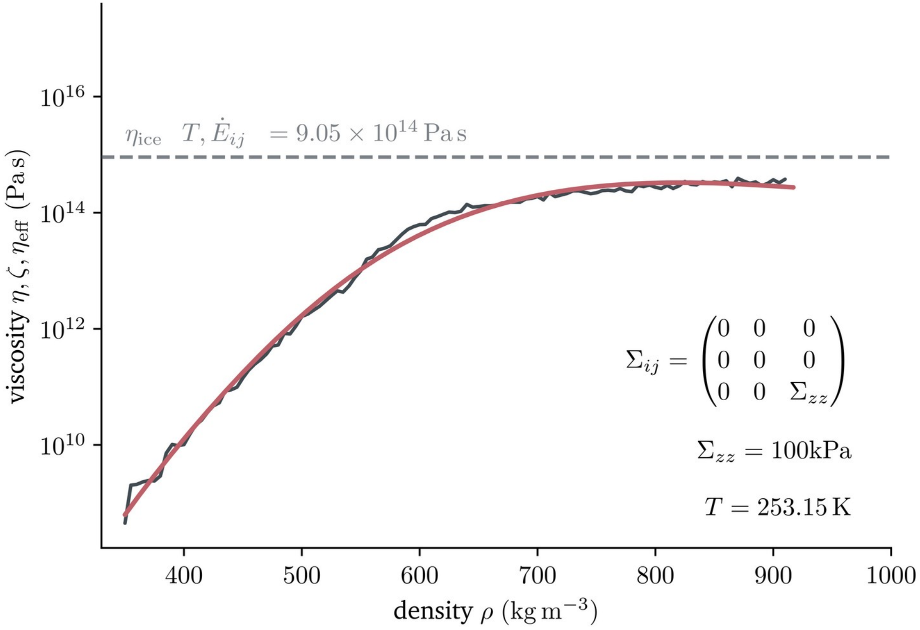


$$\eta\,(\rho, T, \Sigma_{ij}) = \frac{1}{3\,a(\rho)}\,B^*(T)^{-1}\,\left(a(\rho)\,\Sigma_e^2 + b(\rho)\,\Sigma_m^2\right)^{\frac{(1-n)}{2}}$$

$$\zeta\,(\rho, T, \Sigma_{ij}) = \frac{1}{b(\rho)}\,B^*(T)^{-1}\,\left(a(\rho)\,\Sigma_e^2 + b(\rho)\,\Sigma_m^2\right)^{\frac{(1-n)}{2}}$$

η_{ice}

Resulting Material Behavior



$$\eta\left(\rho, T, \Sigma_{ij}\right) = \frac{1}{3\,a(\rho)}\, B^*(T)^{-1} \left(a(\rho)\,\Sigma_e^2 + b(\rho)\,\Sigma_m^2\right)^{\frac{(1-n)}{2}}$$

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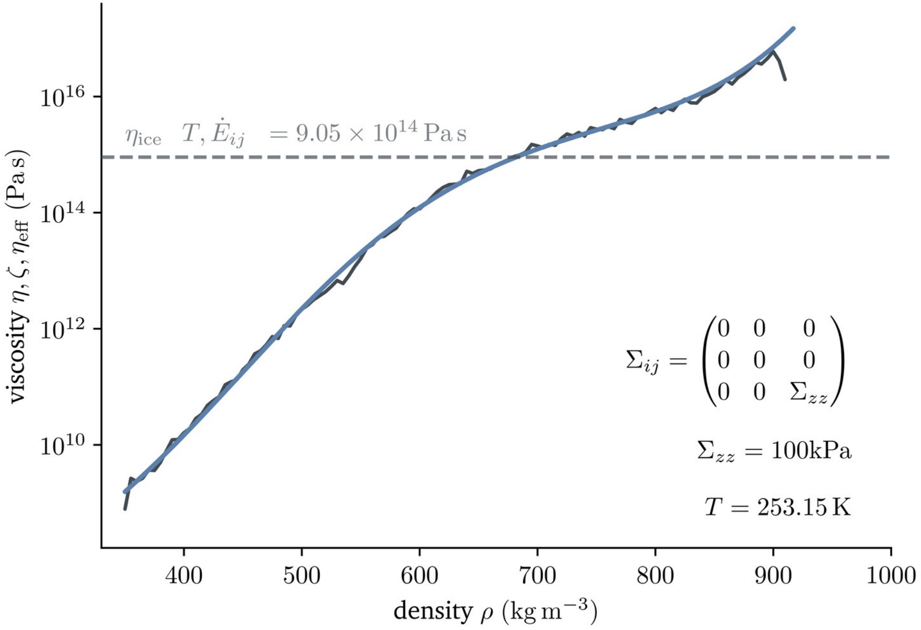
η_{ice}

Resulting Material Behavior



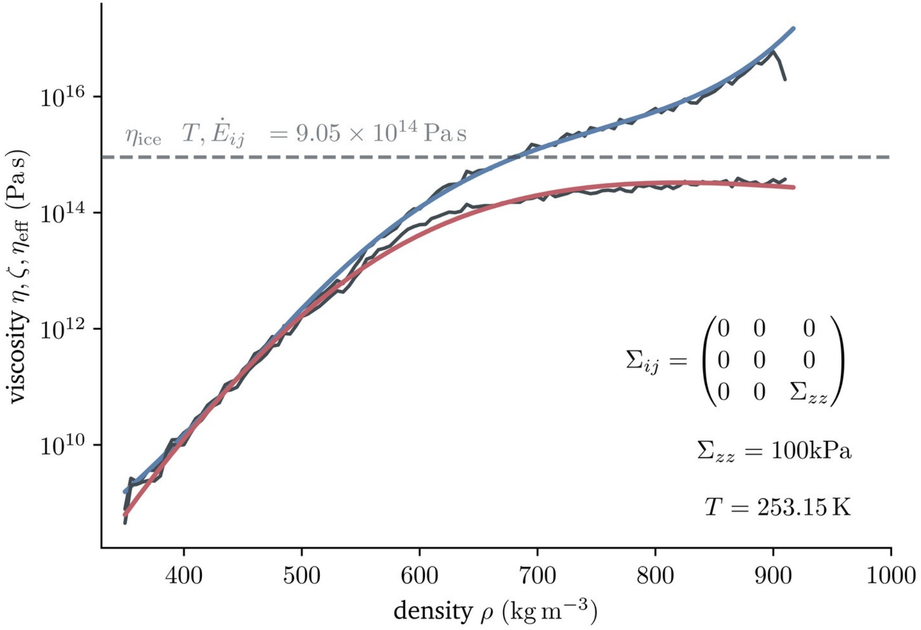
$$\eta\left(\rho, T, \Sigma_{ij}\right) = \frac{1}{3\,a(\rho)} B^*(T)^{-1} \left(a(\rho)\,\Sigma_e^2 + b(\rho)\,\Sigma_m^2\right)^{\frac{(1-n)}{2}}$$

$$\zeta\left(\rho, T, \Sigma_{ij}\right) = \frac{1}{b(\rho)} B^*(T)^{-1} \left(a(\rho)\,\Sigma_e^2 + b(\rho)\,\Sigma_m^2\right)^{\frac{(1-n)}{2}}$$

η_{ice}

Timm Schultz, Angelika Humbert, and Ralf Müller

Resulting Material Behavior

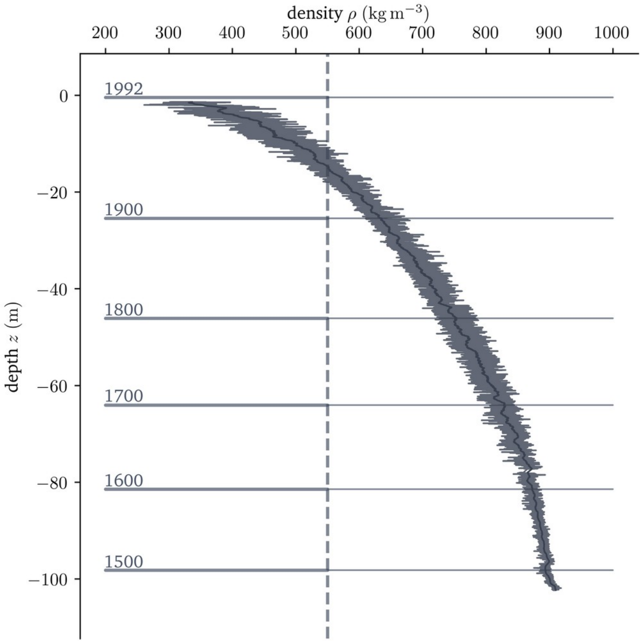


$$\eta\left(\rho, T, \Sigma_{ij}\right) = \frac{1}{3\,a(\rho)} B^*(T)^{-1} \left(a(\rho)\,\Sigma_e^2 + b(\rho)\,\Sigma_m^2\right)^{\frac{(1-n)}{2}}$$

$$\zeta\left(\rho, T, \Sigma_{ij}\right) = \frac{1}{b(\rho)} B^*(T)^{-1} \left(a(\rho)\,\Sigma_e^2 + b(\rho)\,\Sigma_m^2\right)^{\frac{(1-n)}{2}}$$

η_{ice}

Examples - B16



Wilhelms, F. (1996). *Density of ice core ngt03C93.2 from the North Greenland Traverse.* PANGAEA.
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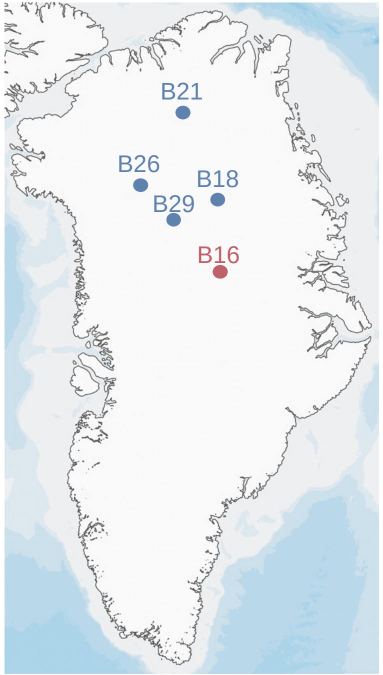
Herron & Langway (1980)

Cell Model Approach



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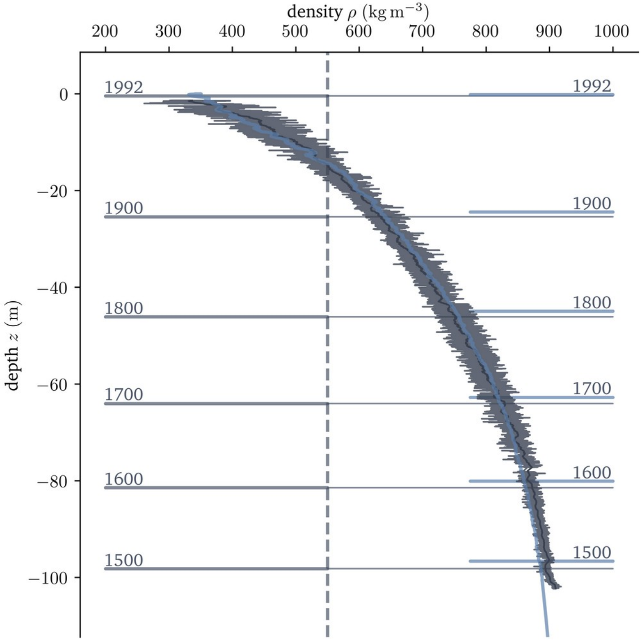
Examples - B16



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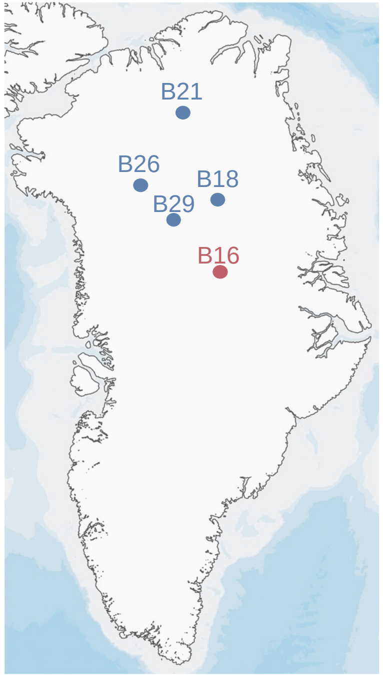
Herron & Langway (1980)

Cell Model Approach



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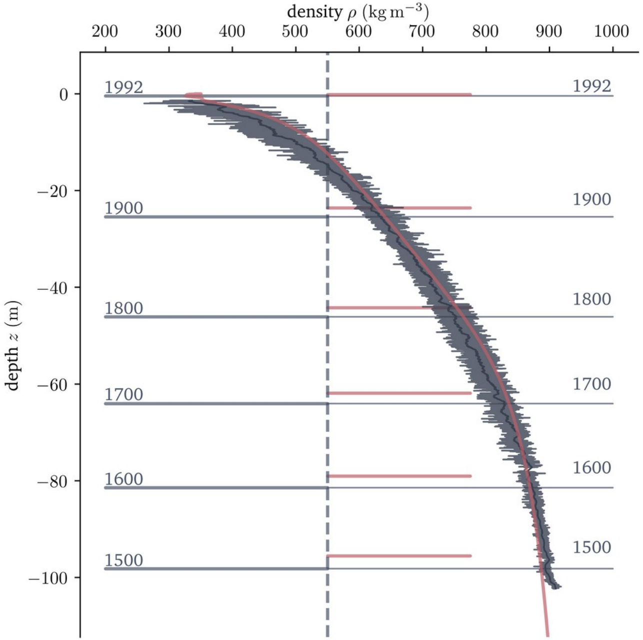
Examples - B16



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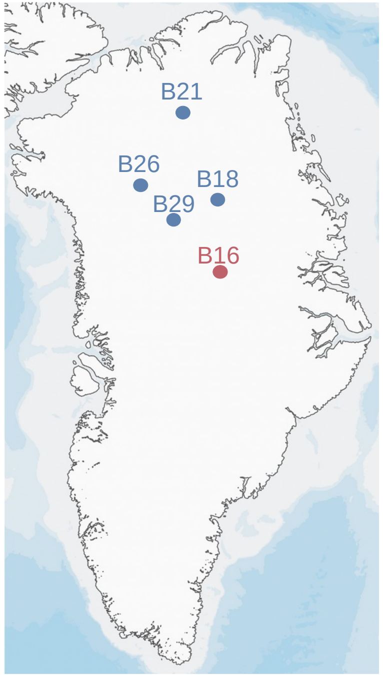
Herron & Langway (1980)

Cell Model Approach



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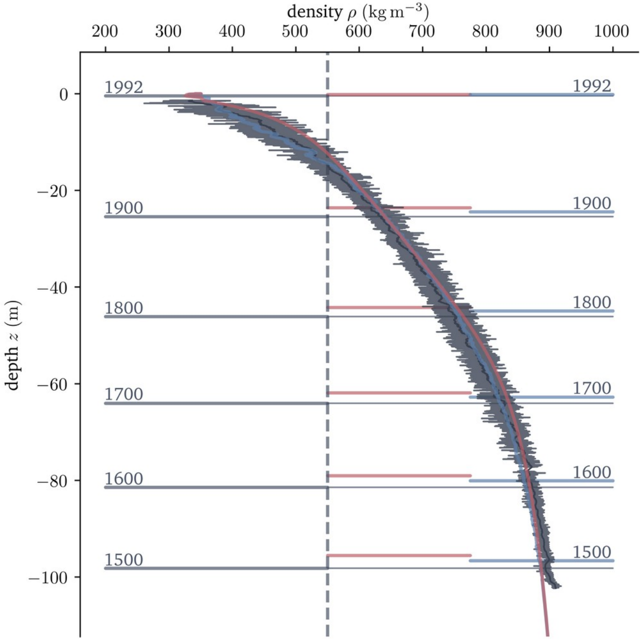
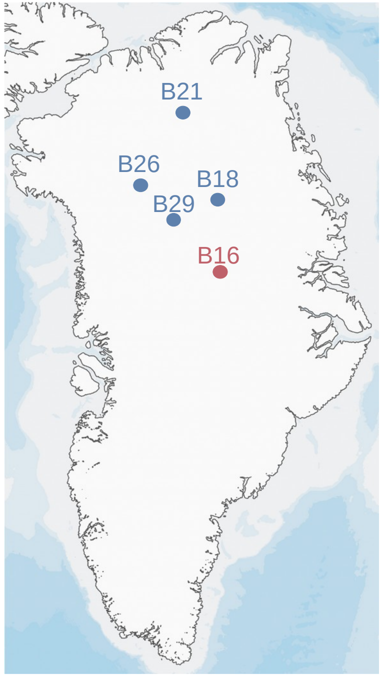
Examples - B16



Herron & Langway (1980)

Cell Model Approach



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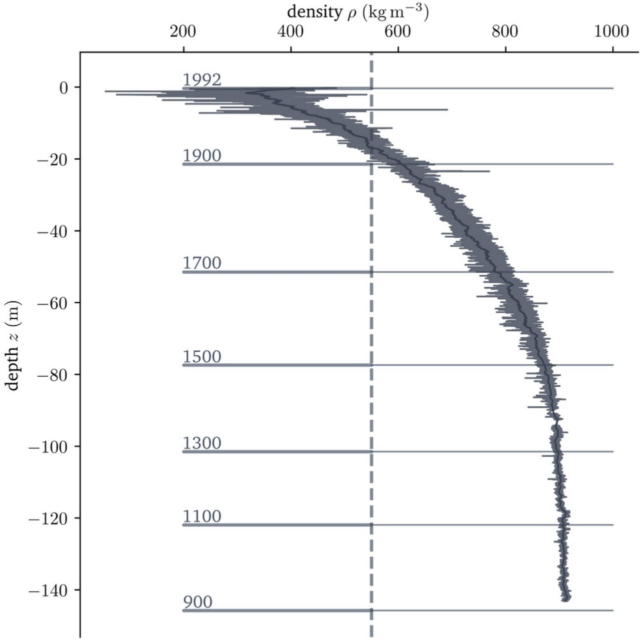
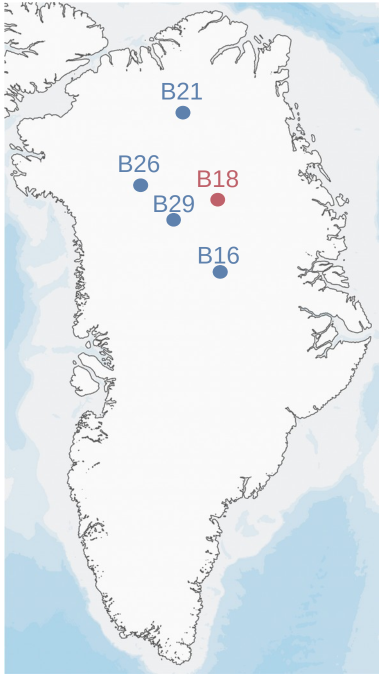
Examples - B18



Herron & Langway (1980)

Cell Model Approach



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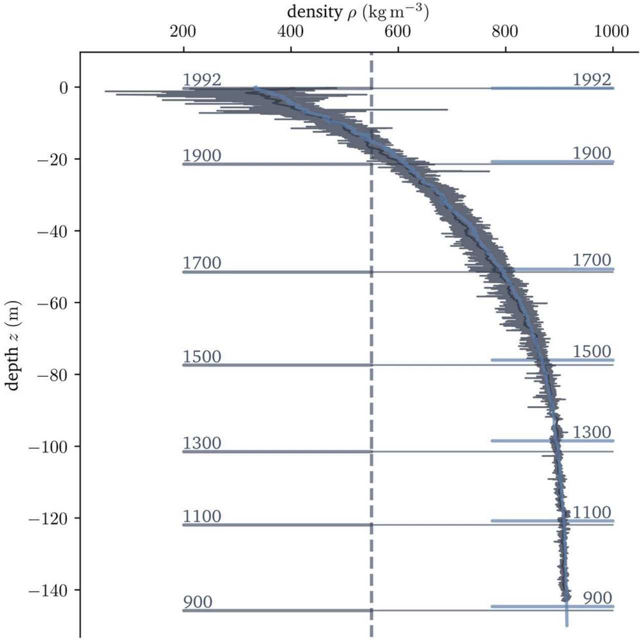
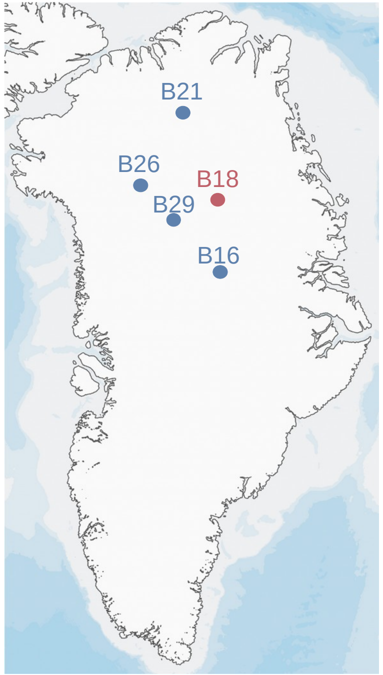
Examples - B18



Herron & Langway (1980)

Cell Model Approach



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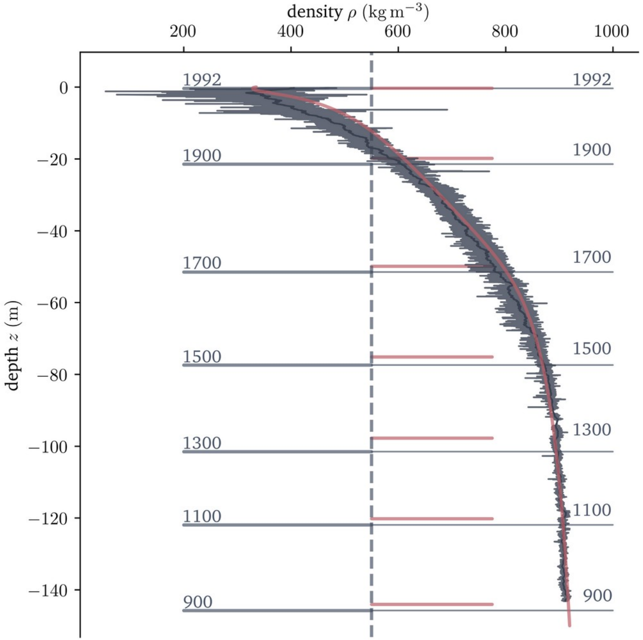
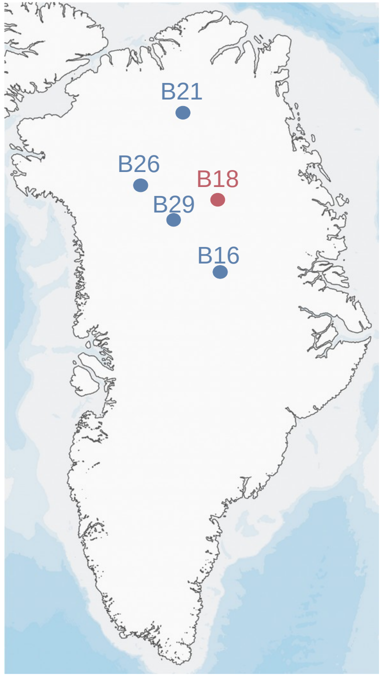
Examples - B18



Herron & Langway (1980)

Cell Model Approach



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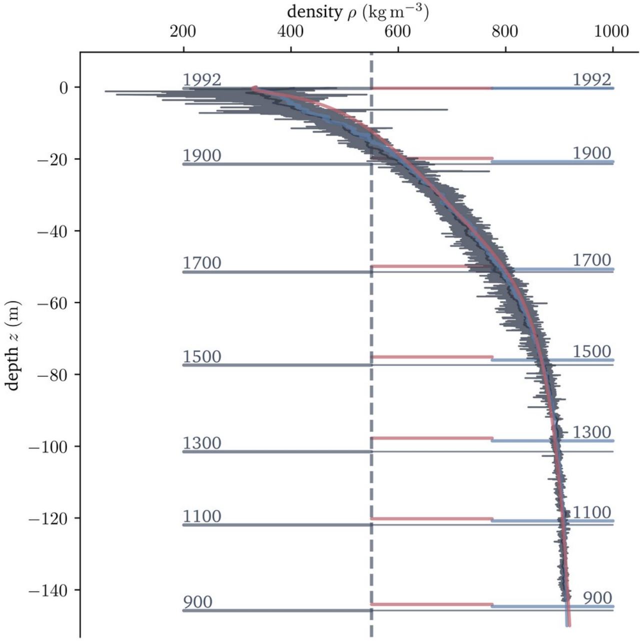
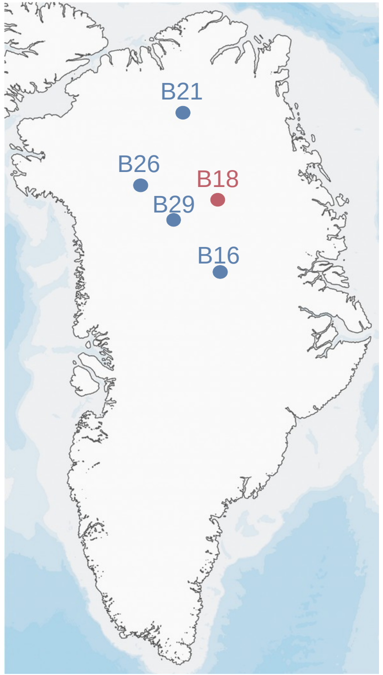
Examples - B18



Herron & Langway (1980)

Cell Model Approach



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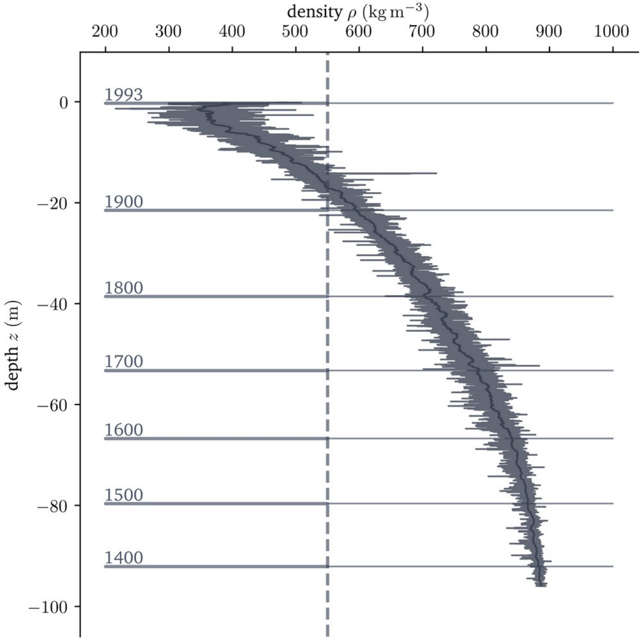
Examples - B21



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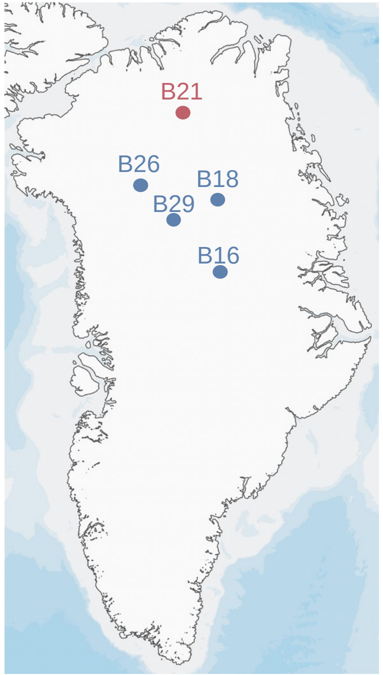
Herron & Langway (1980)

Cell Model Approach



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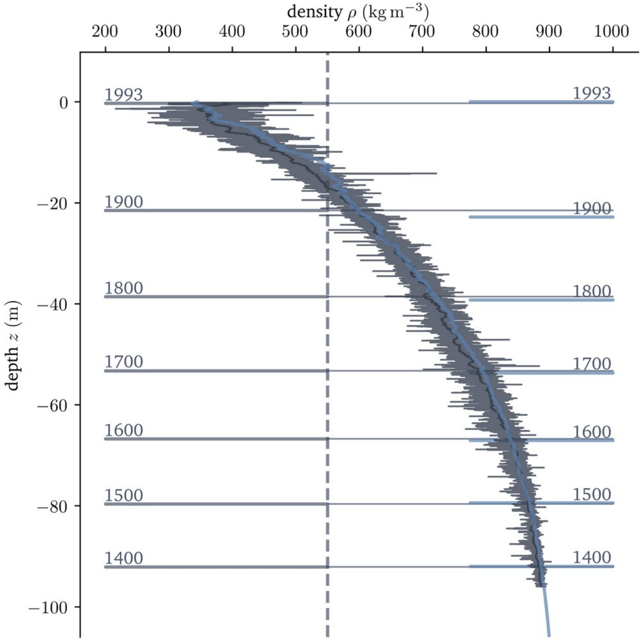
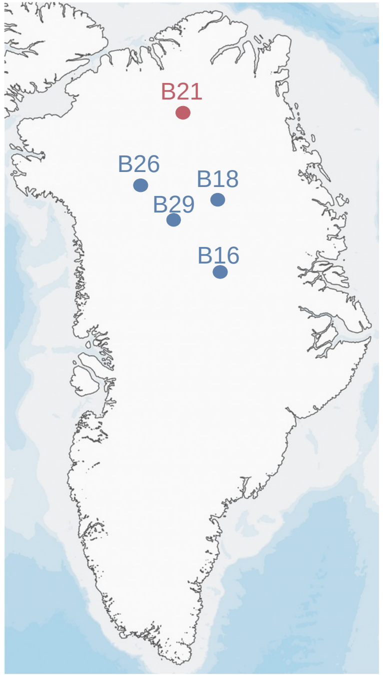
Examples - B21



Herron & Langway (1980)

Cell Model Approach



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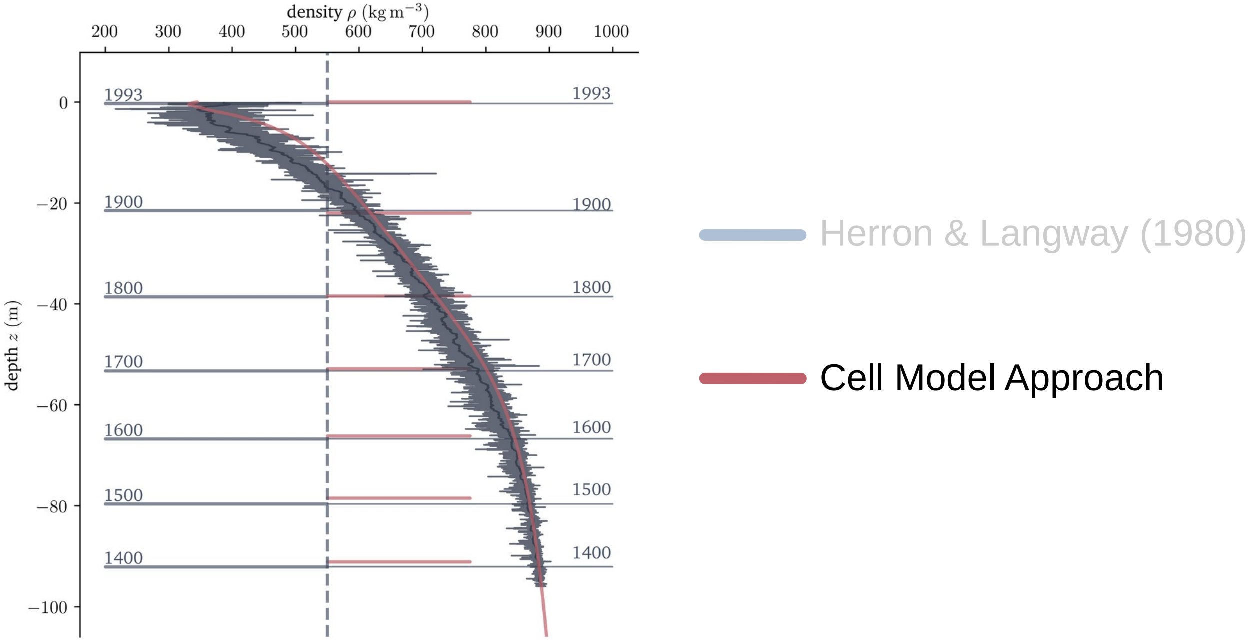
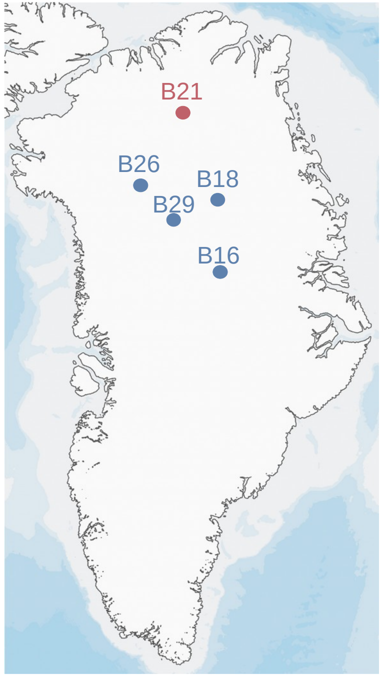
Examples - B21





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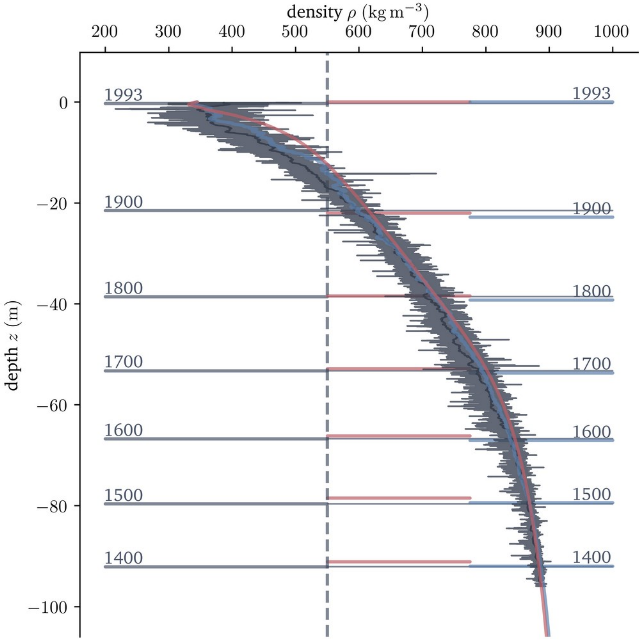
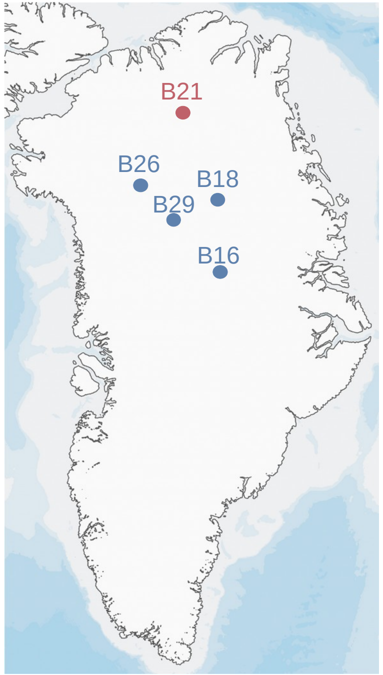
Examples - B21



Herron & Langway (1980)

Cell Model Approach



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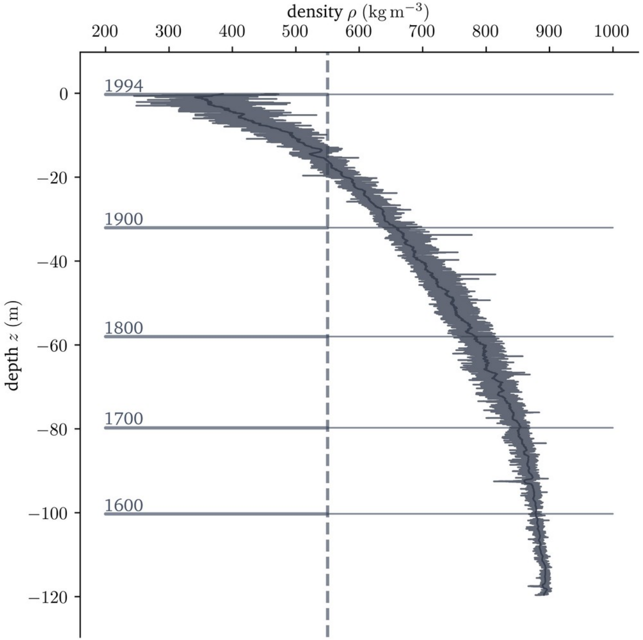
Examples - B26



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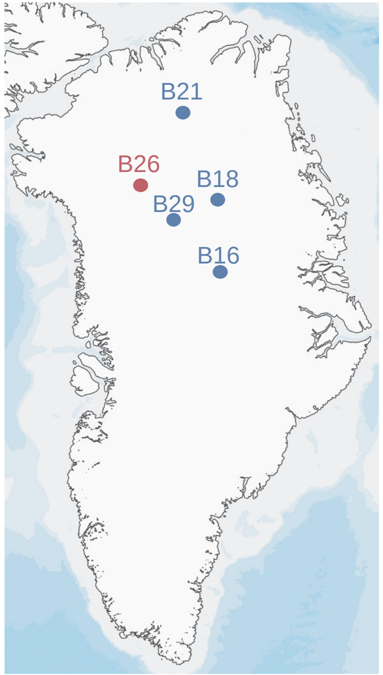
Herron & Langway (1980)

Cell Model Approach



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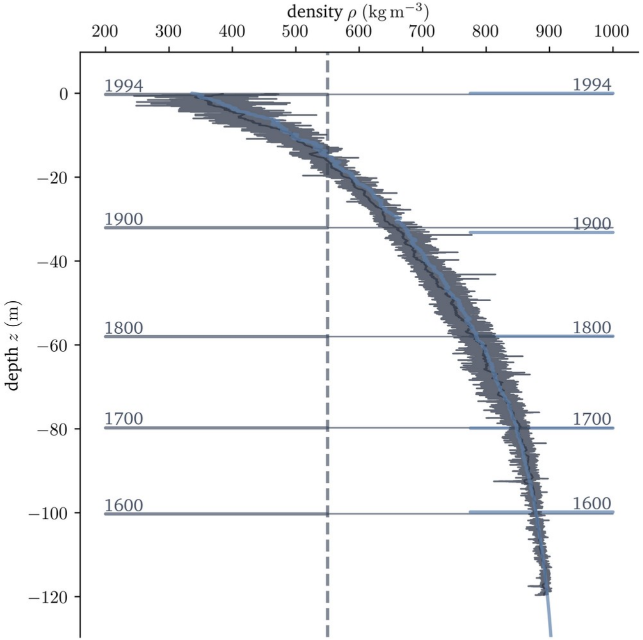
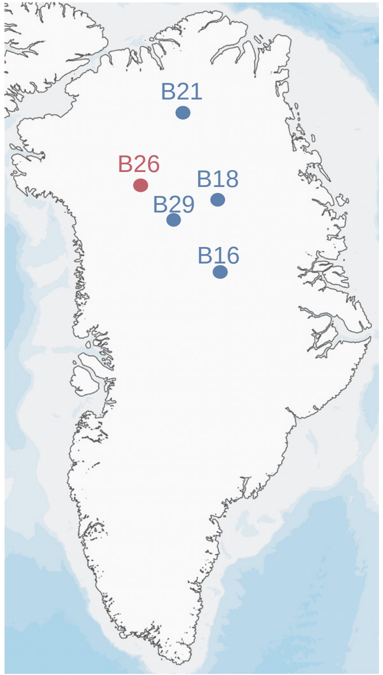
Examples - B26



Herron & Langway (1980)

Cell Model Approach



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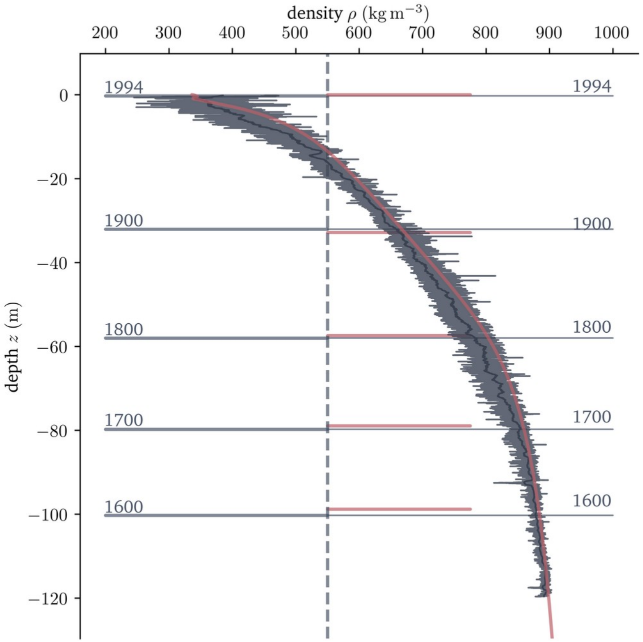
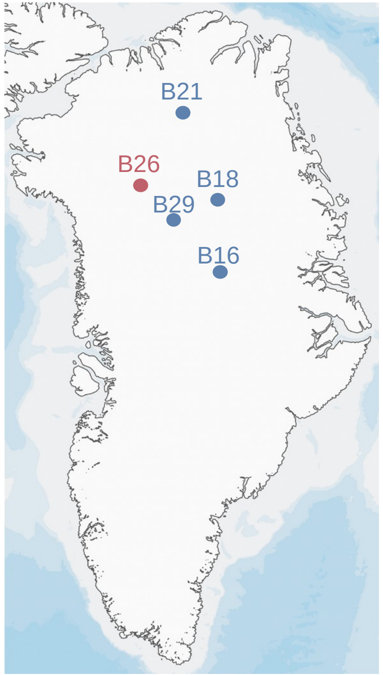
Examples - B26



Herron & Langway (1980)

Cell Model Approach



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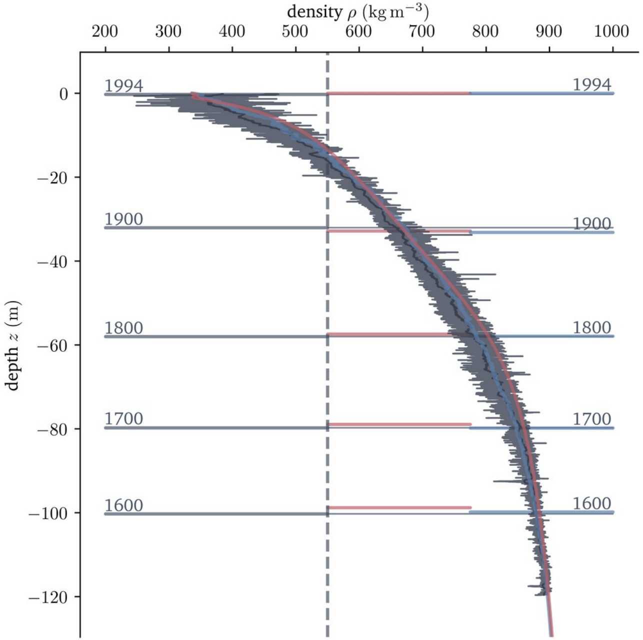

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Herron & Langway (1980)

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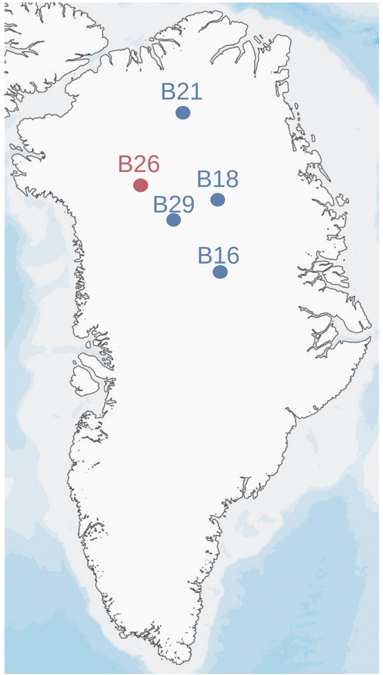

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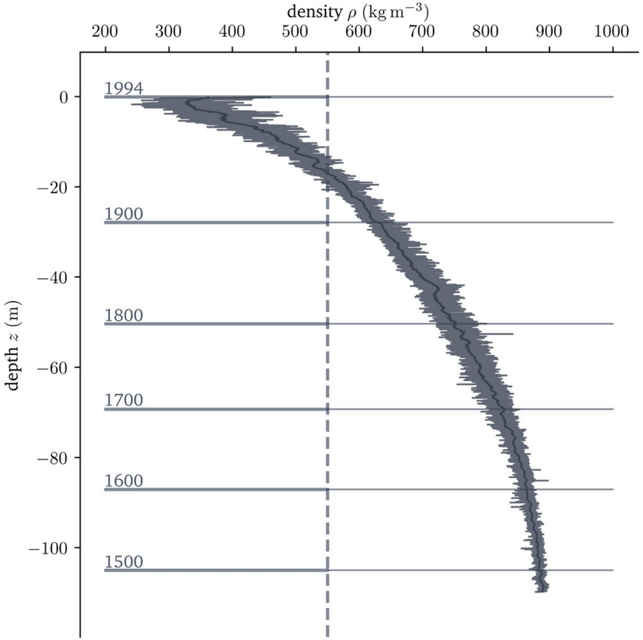
Examples - B29



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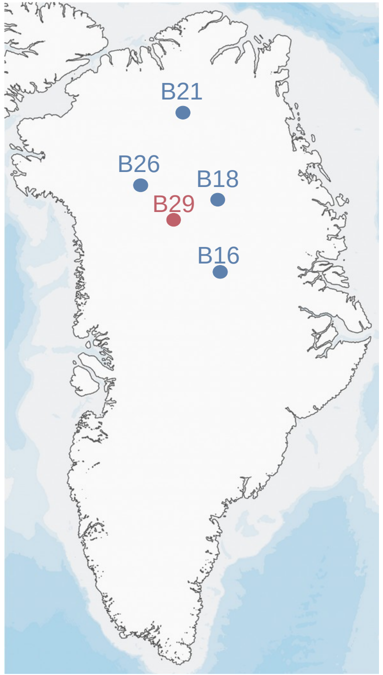
Herron & Langway (1980)

Cell Model Approach



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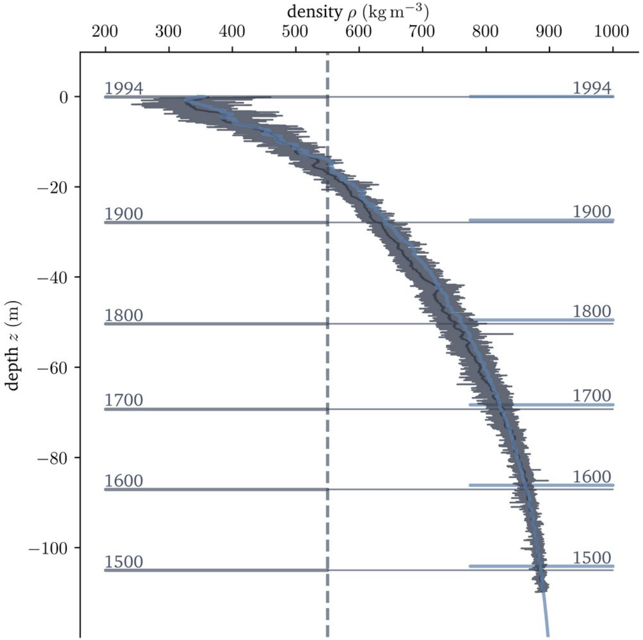

Herron & Langway (1980)

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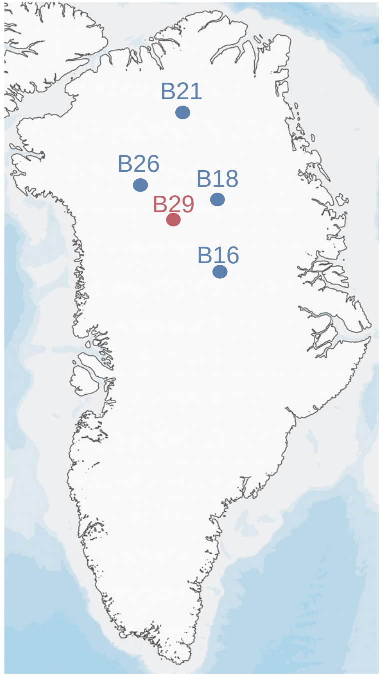

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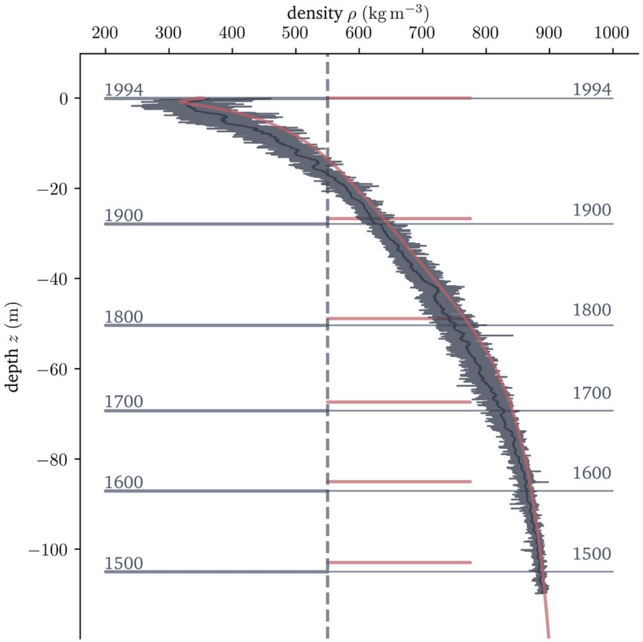

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Herron & Langway (1980)

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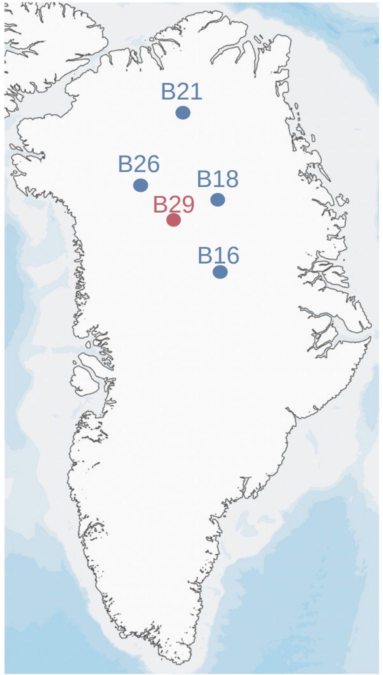

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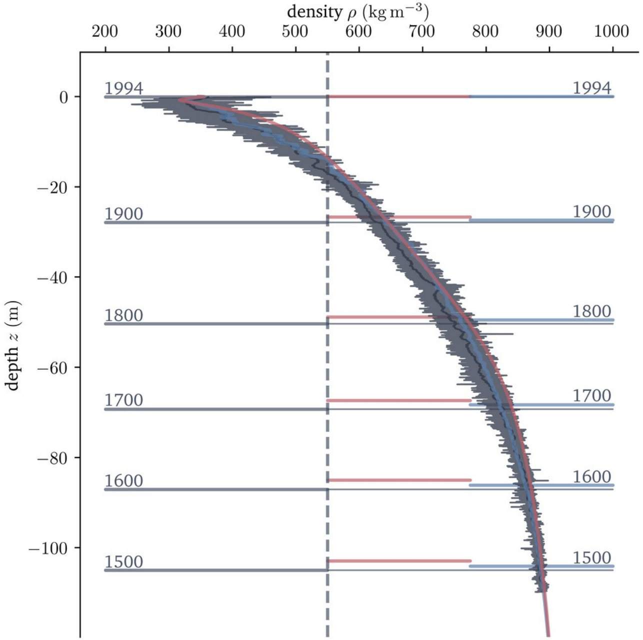

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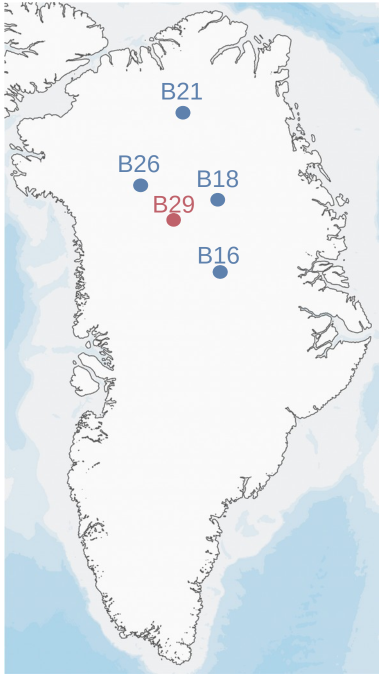

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