How Sensitive are Coarse General Circulation Models to Fundamental Approximations in the Equations of Motion?

Martin Losch[1], Alistair Adcroft, and Jean-Michel Campin

Department of Earth, Atmospheric, and Planetary Sciences, MIT, Cambridge

[1]now at: Alfred-Wegener-Institut für Polar- und Meeresforschung

(email: mlosch@awi-bremerhaven.de)

1 Overview

- Most conventional general circulation models (GCMs) make the Boussinesq approximation, conserving volume instead of mass.
- Is the Boussinesq approximation justified or even necessary? How do the errors incurred compare with those due to the hydrostatic approximation or errors associated with uncertainties in the physical parameterizations? See, for example, McDougall et al. (2002).
- We developed a non-Boussinesq GCM by virtue of the isomorphism of the Boussinesq equations in height coordinates and non-Boussinesq equations in pressure coordinates (see Box 2) in the MIT GCM (Marshall et al., 1997a).
- We compare solutions of non-Boussinesq, Boussinesq, and quasi-hydrostatic models after 1000 years of integration (Boxes 3, 4, and 5).

2 The Isomorphism in the MITgcm

height coordinates (Boussinesq eqs.) $\longleftrightarrow$ **pressure coordinates (non-Boussinesq eqs.)**

dynamical equations

$$-\nabla_z\left(\frac{p}{\rho_0}\right) - f\mathbf{k}\times\mathbf{u} + \mathbf{F} = \frac{D\mathbf{u}}{Dt} \quad\longleftrightarrow\quad \frac{D\mathbf{u}}{Dt} = -\nabla_p\Phi - f\mathbf{k}\times\mathbf{u} + \mathbf{F},$$

$$-g\rho = \frac{\partial p}{\partial z} \quad\longleftrightarrow\quad \frac{\partial\Phi}{\partial p} = -\alpha,$$

$$\nabla_z\cdot\mathbf{u} + \frac{\partial w}{\partial z} = 0 \quad\longleftrightarrow\quad 0 = \nabla_p\cdot\mathbf{u} + \frac{\partial\omega}{\partial p},$$

$$\frac{D\theta}{Dt} = Q \quad\longleftrightarrow\quad \frac{D\theta}{Dt} = Q,$$

$$\frac{DS}{Dt} = Q_S \quad\longleftrightarrow\quad \frac{DS}{Dt} = Q_S,$$

with boundary conditions at the surface ($z = \eta$ and $p = 0$)

$$\frac{D\eta}{Dt} - (P - E) = w \quad\longleftrightarrow\quad \omega = g\rho_{FW}(P - E)$$

with boundary conditions at the bottom ($z = -H(x, y)$ and $p = p_b(x, y)$)

$$-\mathbf{u}\cdot\nabla_z H = w \quad\longleftrightarrow\quad \omega = \frac{\partial p_b}{\partial t} + \mathbf{u}\cdot\nabla_p p_b$$

with

$$\left(\frac{\partial}{\partial t}\right)_z + \mathbf{u}\cdot\nabla_z + w\frac{\partial}{\partial z} = \frac{D}{Dt} \quad\longleftrightarrow\quad \frac{D}{Dt} = \left(\frac{\partial}{\partial t}\right)_p + \mathbf{u}\cdot\nabla_p + \omega\frac{\partial}{\partial p},$$

$$w = \frac{Dz}{Dt} \quad\longleftrightarrow\quad \frac{Dp}{Dt} = \omega$$

(de Szoeke and Samelson, 2002, Marshall et al., Climate modeling exploiting atmosphere-ocean fluid isomorphisms, in preparation)

3 Boussinesq Effects on the General Circulation: SSH Variability

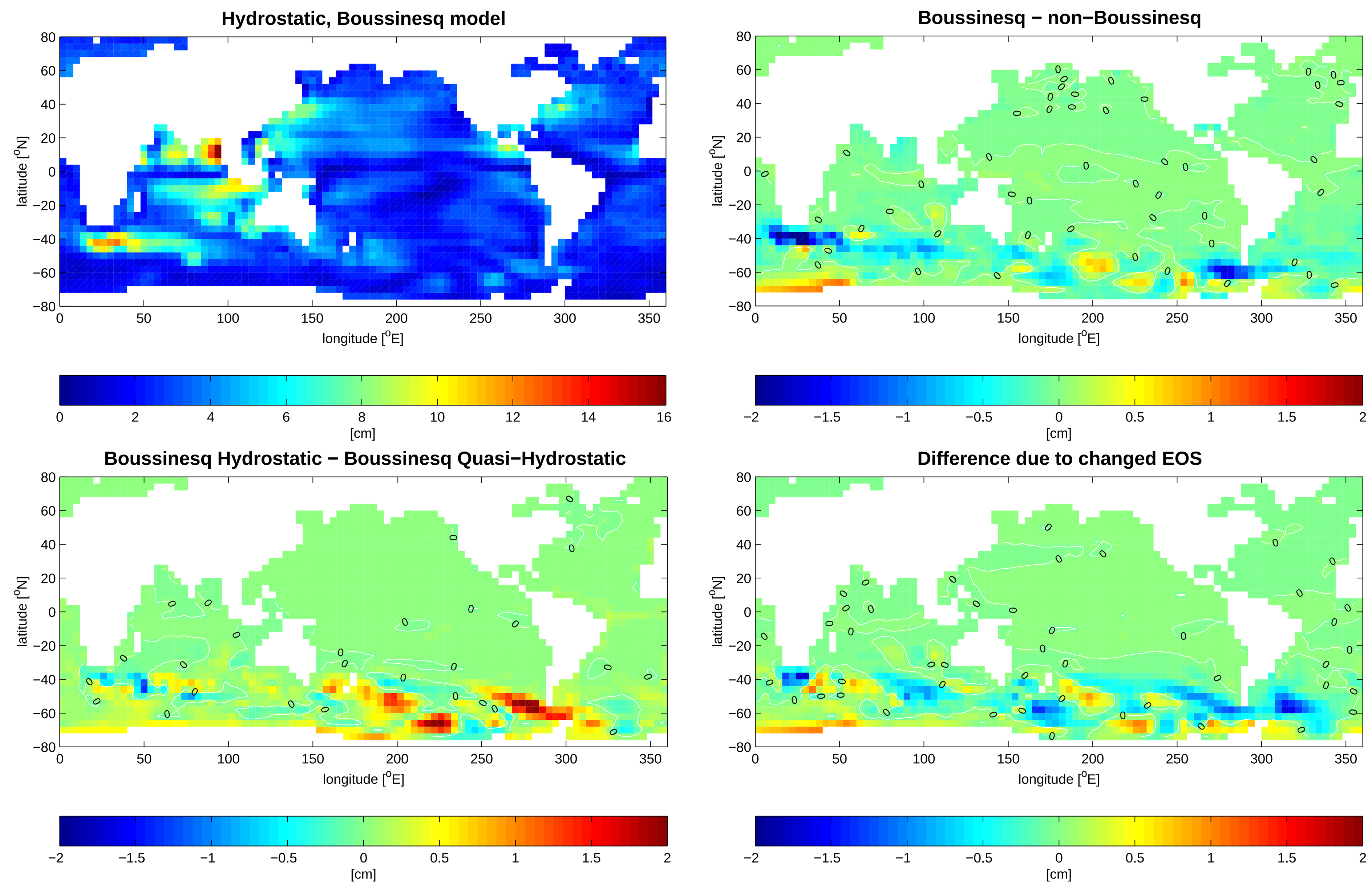

Figure 1: Top left: sea surface height variability (square root of the variance over 100 years in centimeters) of the hydrostatic, Boussinesq model. Top right: difference of sea surface height variability between the Boussinesq and the non-Boussinesq model. Bottom left: change of sea surface height variability when some non-hydrostatic terms in the horizontal momentum equations and the hydrostatic equation have been included. In the terminology of Marshall et al. (1997b), this is a quasi-hydrostatic model. Bottom right: change in sea surface height variability due to the use of a different implementation of the equation of state; Jackett and McDougall (1995) vs. McDougall et al. (2003). *Clearly, the different equation of state changes the sea surface height variability as much as relaxing either the Boussinesq or the hydrostatic approximation.*

4 Comparison of Bottom Pressure Variability

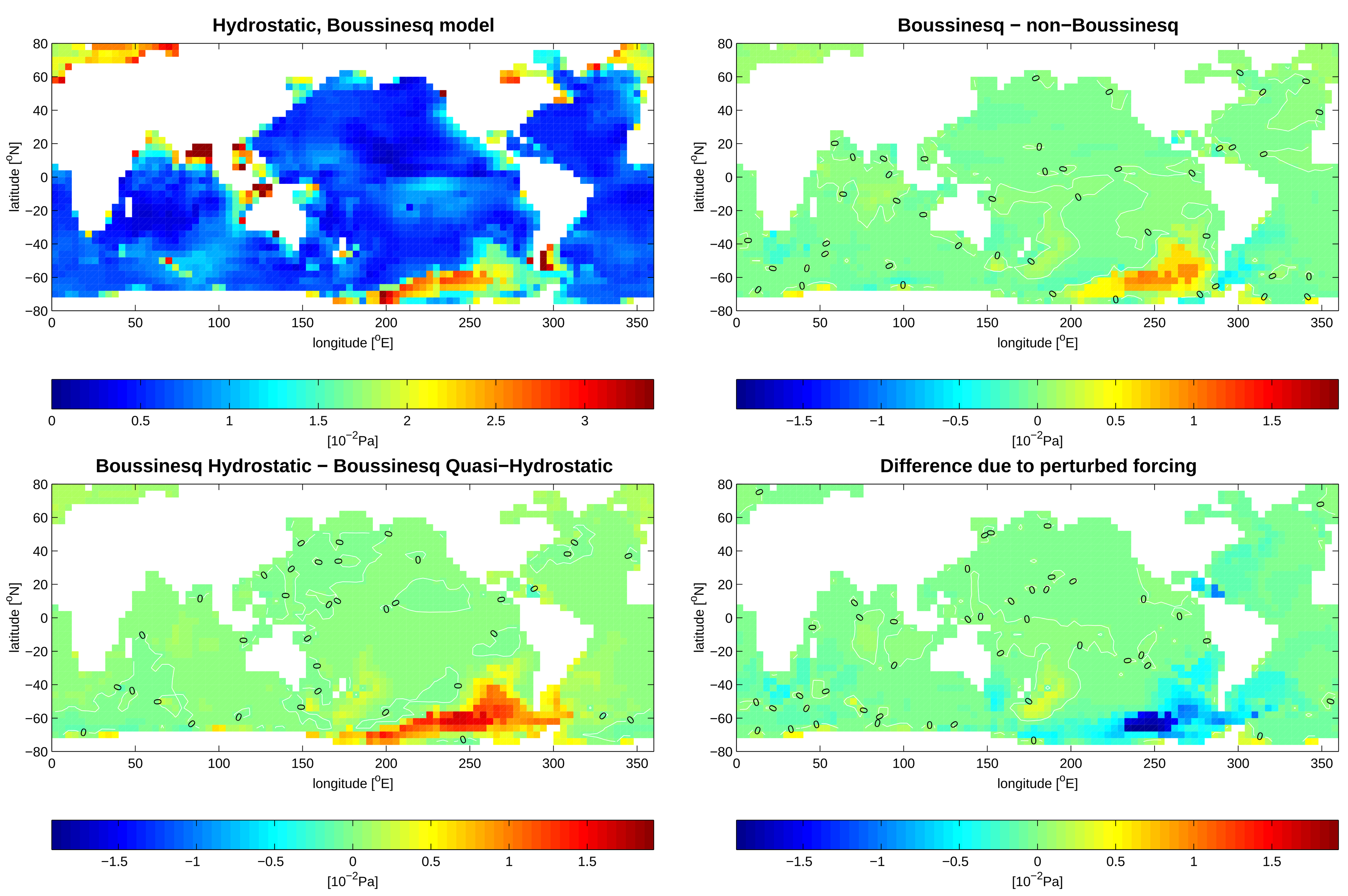

Figure 2: Top left: Bottom pressure variability (square root of the bottom pressure variance of 100 years of integration) of the hydrostatic, Boussinesq model in 10Pascal ≈ 1mm. The model exhibits strong variability in the Pacific sector of the Southern Ocean and in shallow regions. Top right (Boussinesq vs. non-Boussinesq): difference of bottom pressure variability between the height coordinate model and the pressure coordinate model. Bottom left (hydrostatic vs. quasi-hydrostatic): difference between the hydrostatic, Boussinesq model and a model where some of the non-hydrostatic terms in the horizontal momentum equations and the hydrostatic equation have been included. In the terminology of Marshall et al. (1997b), this is a quasi-hydrostatic model. Bottom right: difference in bottom pressure variance after adding random noise of amplitude 2.22×10^{-16} (changing the last digits of a double precision value) to the forcing fields. *Clearly, the changes due to the different model formulations are barely decernable from the effects of numerical round-off.*

5 Relevance to Sea Level Change and Gravity Missions

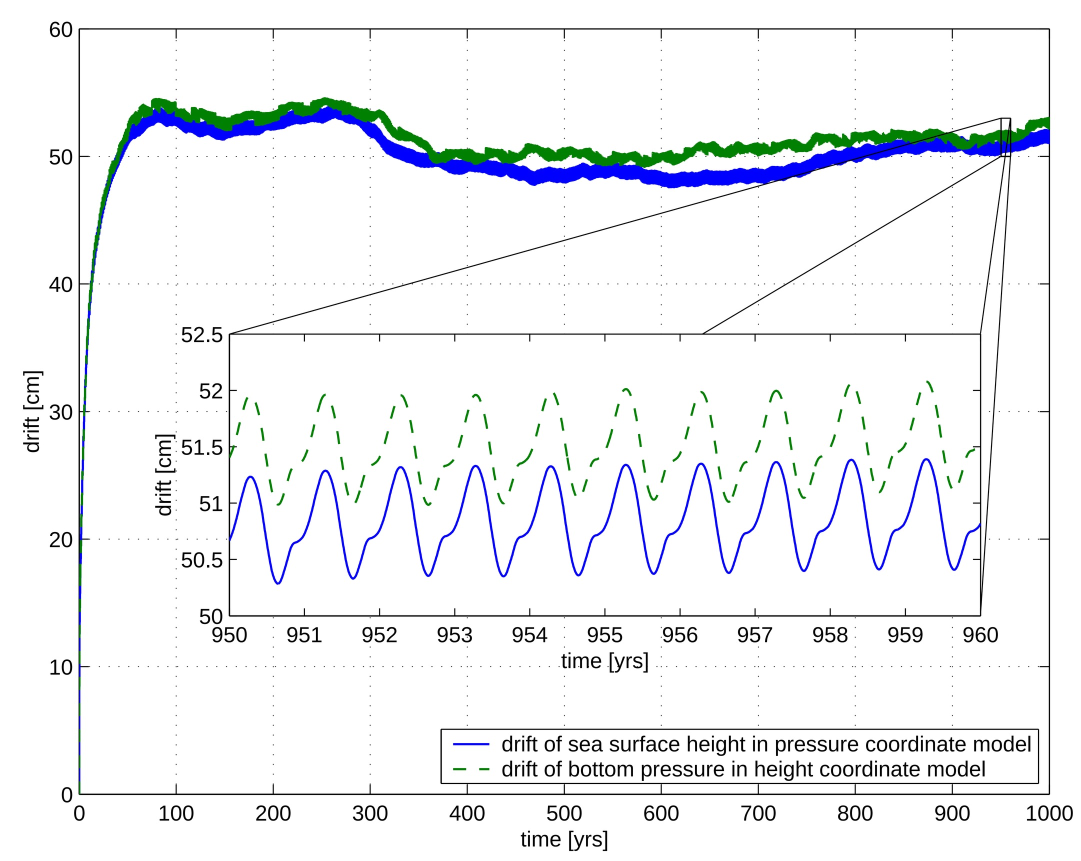

Figure 3: Mass drift of the height coordinate model and volume drift of the pressure coordinate model, scaled to units of centimeters. The Boussinesq models are volume but not mass conserving and therefore the global mean bottom pressure drifts in time. The non-Boussinesq model in pressure coordinates is mass conserving and recovers a global volume drift caused by steric effects. Clearly, the mass drift of the Boussinesq model can be transformed into a volume drift that is remarkably similar to that of the non-Boussinesq model.

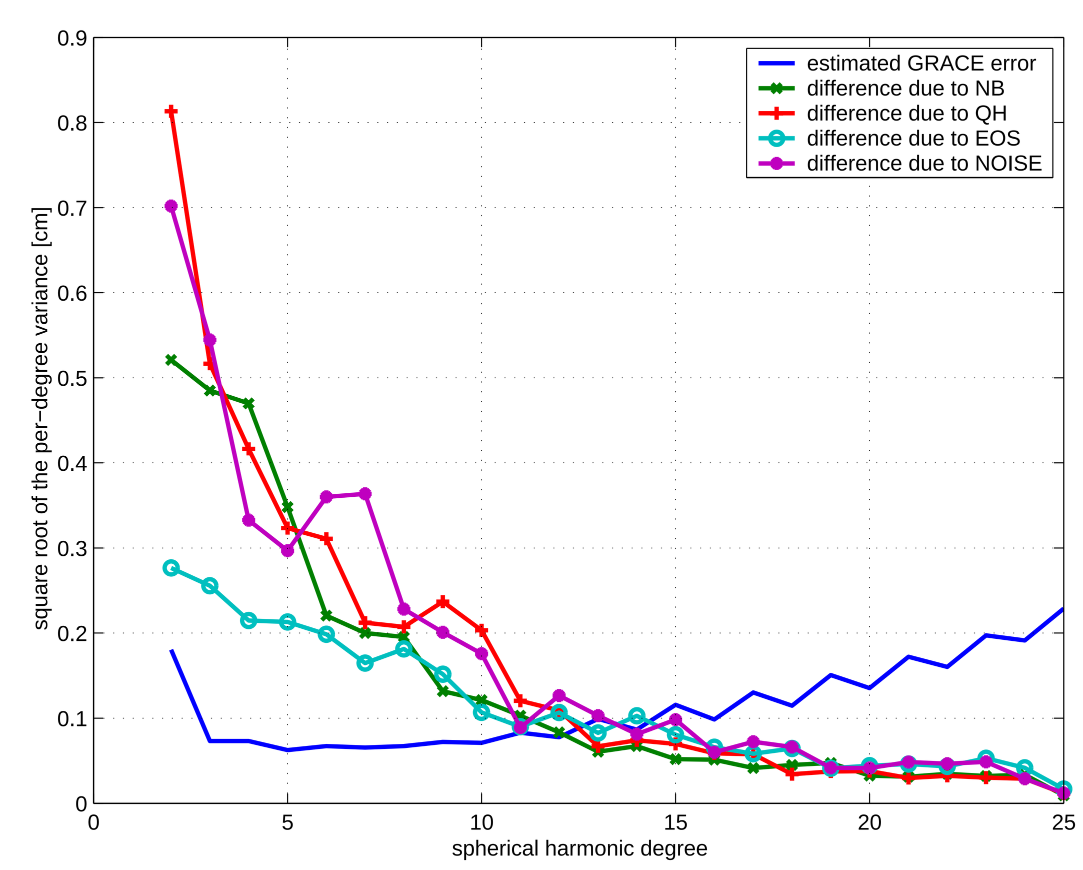

Figure 4: The difference in bottom pressure variability as a function of scale. Shown are the per-degree variances $\sqrt{\sum_m |c_{nm}|^2}$ of the spherical harmonic coefficients c_{nm}. All approximations/errors give rise to differences in bottom pressure variability that exceed the estimated errors of a geoid derived from GRACE (Balmino et al., 1998) at large scales. But Boussinesq effects (NB) seem to be as important as the hydrostatic approximation (QH), small differences in the equation of state (EOS), and numerical noise in the forcing fields (NOISE).

6 Conclusions

- Conventional GCMs make a number of approximations that influence their solution, such as the hydrostatic approximation and the Boussinesq approximations. We find that relaxing the hydrostatic approximation has a larger impact on a coarse resolution global model than do Boussinesq effects.
- Small changes in other approximations, such as the exact form of the equation of state, in physical parameterisations, and numerical noise lead to changes in the circulation, that are at least of the same order of magnitude as those due to Boussinesq effects.
- Because there is no additional cost involved in running a pressure coordinate model, ocean models should be non-Boussinesq. But as far as accuracy is concerned, the Boussinesq approximation is only one of many approximations, and it is certainly not the most severe one.
- **Two Caveats:**
  - Bottom pressure in pressure coordinates is a prognostic variable, in height coordinates it is diagnostic. Diagnostic variables tend to exhibit greater variability, thus biasing the results.
  - Details of the comparison are incomplete. For example, the vertical viscosity and diffusivity in both models are slightly different for technical reasons. This may be the largest contribution to the current differences between the Boussinesq and non-Boussinesq model.

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